Journal of Kufa for Mathematics and Computer Vol.1, No.5, may, 2012, pp.38-43

Construction of Maximum(k,n)-Arcs from(k,m)-Arcs in PG (2,5) for m < n

Sawsan Jawad Kadhum / University of Baghdad, College of Education, Ibn Al-Haitham, Dept.of Mathematics.

Rasha Naser Majeed / University of Baghdad, College of Education, Ibn Al-Haitham, Dept.of Mathematics.

Abstract

The purpose of this work is to find maximum (k, n)-arcs from maximum (k, 2)-arcs where n=3,4,5.in the projective plane PG (2,5).

1.Introduction

Mohammed, 1988[7], showed the classification of (k,3)-arcs in PG(2,5), also Abdul-Hussain, (1997), [1], showed the classification of (k,4)-arcs in PG (2,5), and Faiyad, (2000), [2], constructed and classified (k,3)-arcs in PG (2,7), all of them used the algebraic method. Finally, Ban, (2001), [4], constructed (k,4)-arcs in PG (2,11) by adding two points to each line of the type 2–secant, this means that the addition is horizontally.

In this work we introduce a method to find maximum (k, n)-arcs in PG(2,5), by adding points of index zero to the union of the maxim (k,2)-arcs, this means that the addition is vertically.

2. preliminaries

2.1 Definition [6]

A Projective plane PG (2,5) over GF (5) consists of 31 points, 31 lines, each line contains 6 points and through every point there are 6 lines. Let P_i and L_i , i=1,2,…,31 be the points and the lines of PG (2,5) respectively.

(where P^2+P+1 is the equation which gives the numbers of points and lines in PG (2,5) over GF (5) [2,8])

Let i stands for the points P_i .The points and lines of PG (2,5) are given in table (1) .

2.2 Definition [6]

A (k,n)-arc in PG (2,P) is a set of K points no n+1 of them are collinear.

A (k,2)-arc is called k-arc which is a set of K points no three of them are collinear.

A (k,n)-arc is complete if it is not contained in a (k+1,n)-arc .The maximum number of points that a (k,2)-arc can have is m (2,p), and this arc is an oval .

2.3 Theorem [6]

$$m(2,p)=\begin{cases} p+1 & \text{for } p \text{ odd} \\ p+2 & \text{for } p \text{ even} \end{cases}$$

2.4 Definition [3,5]

A line L in PG (2,P) is an i-secant of a (k, n)-arc if $|L \cap K|$=i.

A 2-secant is called a bisecant line.

2.5 Definition [3]

A variety V (F) of PG (2,P) is a subset of PG (2,P) s.t.

$V(F) = \{ P(A) \in PG(2,P) \mid F(A) = 0 \}$

2.6 Definition [6]

Let Q (2,P) be the set of quadrics in PG (2,P), that is the varieties V (F), where:

$F = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$

If V(F) is non singular, then the quadric is a conic.

2.7 Theorem [5]

Every conic in PG (2,P) is a (P+1)-arc.

2.8 Theorem [5]

In PG (2,P), with P odd, every oval is a conic.

2.9 Definition [3]

A point N which is not on a (k, n)-arc has index i if there are exactly i (n-secants) of the arc through N, we denote the number of points N of index i by N_i .

2.10 Remark [5]

The (k, n)-arc is complete iff N0=0. Thus the arc is complete iff every point of PG (2,p) lies on some n-secant of the arc.

3.The Construction of (k,2)-Arcs in PG(2,5)

Let A={1,2,7,13} be the set of reference and unit points where:-

1=(1,0,0), 2=(0,1,0), 7=(0,0,1), 13= (1,1,1).

A is a (4,2)-arc since no three points of A are collinear. There are six points of index zero for A, which are: 20,21,24,26,29,30.Hence A is incomplete (4,2)-arc.

3.1 The Conics in PG (2,5) Through The Unit and Reference Points [6]

The general equation of the conic is:

$a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_3 = 0$ ……………………… (1)

By substituting the points of A in (1), we get:

$a_1 = a_2 = a_3 = 0$

And

$a_4 + a_5 + a_6 = 0$

so (1) becomes:

$a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_3 = 0$ ………………………………………………..(2)

If $a_4=0$,Then the conic is degenerated. Therefore $a_4 \neq 0$, similarly $a_5 \neq 0$ and $a_6 \neq 0$.

Dividing equation (2) by a_4, we get:

$x_1x_2 + \alpha x_1x_3 + \beta x_2x_3 = 0$

Where $\alpha = \frac{a_5}{a_4}$ and $\beta = \frac{a_6}{a_4}$, then $\beta = -(1+\alpha)$ since $1+\alpha+\beta = 0 \pmod 5$.

$\alpha \neq 0$ and $\alpha \neq 4$ for if $\alpha = 0$ or $\alpha = 4$,we get a degenerated conic, i.e., $\alpha = 1,2,3$ and (2) can be written as:

$x_1x_2 + \alpha x_1x_3 - (1+\alpha) x_2x_3 = 0$ ………………………………………………(3)

3.2The Equations and The Points of PG(2,5) Through The Unit and Reference Points [6]

For any value for α there is a unique conic contains six points.

1. If α =1,then the equation of the conic C_1 is $x_1x_2 + x_1x_3+3x_2x_3 =0$
   The points of C_1 are $\{1,2,7,13,20,26\}$ which is a complete arc.
2. If α =2, then equation of the conic C2 is x1x2+2x1x3+2x2x3=0 and
   C_2={1,2,7,13,21,29} which is a complete arc.

3. If α =3, then the equation of the conic C_3 is $x_1x_2+3x_1x_3+x_2x_3=0$ and
   C_3={1,2,7,13,24,30} which is a complete arc. Thus we found three maximal
   (k, 2)-arcs C_1, C_2, C_3.

4.The Construction of Maximum (K, 3)-Arcs in PG (3,5)

We will try to get maximum (K, 3)-arcs through the following steps:

a-We take the union of every two maximum (K, 2)-arcs. We see that each set is incomplete (K, 3)-arc. Since there exist points of index zero for every arc, i.e., $C_0 \neq 0$.
b-We get complete arcs by adding the points of index zero to every incomplete arc.
c-We take only the maximum arcs, as the
Let $D_1 = C_1 \cup C_2$
= {1,2,7,13,20,26,21,29}is incomplete.

We add to D_1 three points of index zero, which are {3,9,12}.
Then D_1' ={1,2,7,13,20,26,21,29,3,9,12} is a complete (11,3)-arc, since C_0=0.
Let $D_2 = C_1 \cup C_3$
= {1,2,7,13,20,26,24,30}is incomplete.
We add to D_2 three points of index zero, Which are{4,8,12}.
Then D_2' = {1,2,7,13,20,26,24,30,4,8,12} is a complete (11,3)-arc, since C_0=0.

Let $D_3 = C_2 \cup C_3$
= {1,2,7,13,21,29,24,30}is incomplete.
We add to D_3 three points of index zero, which are {3,8,15}.
Then D_3' = {1,2,7,13,21,29,24,30,3,8,15} is a complete (11,3)-arc, since C_0=0.
Then the maximum (11,3)-arcs in PG (2,5) are :
D_1' = {1,2,7,13,20,26,21,29,3,9,12}
D_2' = {1,2,7,13,20,26,24,30,4,8,12}
D_3' = {1,2,7,13,21,29,24,30,3,8,15}

5.The Construction of Maximum (K, 4)-Arcs in PG (2,5)

We will try to get maximum (K, 4)-arcs through the following step:

a-We take the union of every two maximum (K, 3)-arcs. We see that each set is incomplete (K, 4)-arcs. Since there exist points of index zero for every arc, i.e., $C_0 \neq 0$.
b-We get complete arcs by adding the points of index zero to every incomplete arc.

c-We take only the maximum arcs, as the following:

Let $G_1 = D_1' \cup D_2'$
= {1,2,7,13,20,26,21,29,3,9,12,24,30,4,8}
is incomplete.
We add to G_1 one point of index zero, Which is {15}.

Then G_1' =
{1,2,7,13,20,26,21,29,3,9,12,24,30,4,8,15}
is a incomplete (16,4)-arc since C_0= 0.

Let $G_2 = D_1' \cup D_3'$
={1,2,7,13,20,26,21,29,3,9,12,24,30,8,15}
is incomplete.
We add to G_1 one point of index zero, Which is {4}.
Then
G_2'={1,2,7,13,20,26,21,29,3,9,12,24,30, 8,15,4} is a incomplete (16,4)-arc since C_0=0.

Let $G_3 = D_2' \cup D_3'$
={1,2,7,13,20,26,24,30,4,8,12,21,29,3,15}
is incomplete.
We add to G_3 one point of index zero, Which is {9}.
Then G_3' =
{1,2,7,13,20,26,24,30,4,8,12,21,29,3,15,9}
is a incomplete (16,4)-arc since C_0=0.

We notice that the arcs G_1' ,G_2' and G_3' are the same set denoted by G_9.
i.e ,$G_4 = G_1' = G_2' = G_3'$
Then the maximum (16,4)-arc in PG (2,5) is G_4={G_1}.

6-The Construction of Maximum (K, 5)-Arcs in PG (2,5)

We take from (5) the maximum (16,4)-arc G_4 which is incomplete (K, 5)-arc, Since there exist points of index zero for G_4 which are
{5,6,10,11, 14,16,17,18,19,22,23,25,27,28,31} i.e., $C_0 \neq 0$.
We add to G_4 five points of index zero, Which are {5,10,14,17,22}.
Then
G_5={1,2,7,13,20,26,21,29,3,9,12,24,30,4, 8,15,5,10,14,17,22} is a maximum (21,5)-arc since C_0=0.

Conclusion

1. We can get the maximum (k, n+1)-arcs from the maximum (k,arcs by taking the union of every two maximum (k, n)-arcs,n=2,3.
2. We get a maximum (k, 5)-arc from maximum (k, 4)-arc.
3. We add the points of index zero for each incomplete arc.
4. Notice, when constructing the maximum (k, n)-arcs where $2 \leq n \leq 4$ That the difference between the lengths of maximum (k, n+1)-arcs and maximum (k, n)-arcs is five points, i.e., p=5[the characteristic of PG (2,5)].
5. We see when the maximum (k, n)-arcs exist ,then k=(n-1)p+1,when n=2,3,4,5

Table (1)
Points and lines of PG (2,5)

i	P_i			L_I					
1	1	0	0	2	7	12	17	22	27
2	0	1	0	1	7	8	9	10	11
3	1	1	0	6	7	16	20	24	28
4	2	1	0	4	7	14	21	23	30
5	3	1	0	5	7	15	18	26	29
6	4	1	0	3	7	13	19	25	31
7	0	0	1	1	2	3	4	5	6
8	1	0	1	2	11	16	21	26	31
9	2	0	1	2	9	14	19	24	29
10	3	0	1	2	10	15	20	25	30
11	4	0	1	2	8	13	18	23	28
12	0	1	1	1	27	28	29	30	31
13	1	1	1	6	11	15	19	23	27
14	2	1	1	4	9	16	18	25	27
15	3	1	1	5	10	13	21	24	27
16	4	1	1	3	8	14	20	26	27
17	0	2	1	1	17	18	19	20	21
18	1	2	1	5	11	14	17	25	28
19	2	2	1	6	9	13	17	26	30
20	3	2	1	3	10	16	17	23	29
21	4	2	1	4	8	15	17	24	31
22	0	3	1	1	22	23	24	25	26
23	1	3	1	4	11	13	20	22	29
24	2	3	1	3	9	15	21	22	28
25	3	3	1	6	10	14	18	22	31
26	4	3	1	5	8	16	19	22	30
27	0	4	1	1	12	13	14	15	16
28	1	4	1	3	11	12	18	24	30
29	2	4	1	5	9	12	20	23	31
30	3	4	1	4	10	12	19	26	28
31	4	4	1	6	8	12	21	25	29

بناء الأقواس-(k,n) العظمى من الاقواس-(k,m) في المستوى الاسقاطي PG(2,5) حيث m<n

سوسن جواد كاظم/ جامعة بغداد – كلية التربية ابن الهيثم – قسم الرياضيات

رشا ناصر مجيد / جامعة بغداد – كلية التربية ابن الهيثم – قسم الرياضيات

المستخلـــــص

الغرض من هذا البحث هو لإيجاد أقواس-(k,n)عظمى من أقـواس-(k,2)عظمى،حيث أن n=3,4,5 فـي المستوى الاسقاطي(PG(2,5.

REFERENCES

[1]. Abdul-Hussain, M.A.,(1997),"Classification of (k, 4)-Arcs in the Projective Plane of Order Five ", M.Sc.Thesis, University of Basrah, Iraq .

[2]. Faiyad ,M.S.,(2000)," Classification and Construction of (k, 3)-Arcs on Projective Plane Over Galois Field GF(7) ",M.Sc.Thesis, University of Baghdad, Iraq .

[3]. Hassan ,A.S.,(2001),"Construction of (k,3)-Arcs in Projective Plane Over Galois Field GF(q),$q=p^h$,when P=2 and h=2,3,4 ", M.Sc.Thesis, University of Baghdad, Iraq .

[4]. Kassim, B.A, (2001)" Upper bound for (k, n)-Arcs ", M.Sc.Thesis, University of Mosul, Iraq .

[5]. Kadhum ,R.,S.,(1997),"Classification of K-Arcs in Projective Plane Over Galois Field ",M.Sc.Thesis, Saddam University, Baghdad, Iraq.

[6]. Kadhum, S.J.,(2001),"Construction of (k,n)-Arcs in PG (2,p) for $2 \le m < n$ " M.Sc.Thesis, University of Baghdad, Iraq .

[7]. Mohammed, M.,J., (1988),"Classification of (k, 3)-Arcs and (k, 4)- Arcs in Projective Plane Over Galois Field ",M.Sc.Thesis, University of Baghdad .

[8]. Yasin, A.L., (1986), "Cubic Arcs in the Projective Plane of Order Eight ", Ph.D. Thesis , University f Sussex , England .