Fuzzy Semi Compact and Fuzzy Semi Coercive Mappings

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Abstract

In this paper ,we introduce the concept of fuzzy semi compact and fuzzy semi coercive mappings in fuzzy topological spaces ,Wediscuss several characterizations and some interesting properties of these mappings. Also, we explain the relation between fuzzy semi compact mapping and fuzzy semi coercive mapping.

Keywords

semi continues , Fuzzy semicompact space, semi-q-nbd, fuzzy semicluster point.

Introduction

The concept of fuzzy set and fuzzy set operations were first introduced by Zadeh [13] in 1965. Several other authors applied fuzzy sets to various branches of mathematics. One of these objects is a topological space. At the first time in 1968, Chang in [2] formulated the natural definition of fuzzy topology on a set and investigated how some of the basic ideas and theorems of point-set topology behave in generalized setting. Chang's definition on a fuzzy topology is very similar to the general topology by exchange all subsets of a universal set by fuzzy subset but this definition is not investigate some properties if we comparison with the general topology. For example in a general topology, any constant mapping is continuous while this idea is not true in Chang's definition on fuzzy topology. One of the very important concepts in fuzzy topology is the concept of mapping. There are several types of mapping. The purpose of this paper is to introduced and study the concept of fuzzy semi compact and fuzzy semi coercive mappings in fuzzy setting and explain the relation between them.

§ 1. Preliminaries

we present some fundamental definitions and propositions which are needed in the next sections.

***Definition (1.1)* [13]**

Let X be a non-empty set and let I be the unit closed interval, i.e., $I=[0,1]$. A fuzzy set A in X is a function from X into the unit closed interval I (i.e., $A: X \longrightarrow I$ be a function). $A(x)$ is interpreted as the degree of membership of element x in a fuzzy set A for each $x \in X$. A fuzzy set A in X can be represented by the set of pairs: $A=\{(x, A(x)): x \in X\}$. The family of all fuzzy sets in X is denoted by I^X.

***Remark (1.2)*[7]**

(a) The empty fuzzy set is a fuzzy set has membership defined by $0(x) = 0$,for all $x \in X$. The empty fuzzy set on a set X denoted by 0_X.

(b) The universal fuzzy set is a fuzzy set has membership defined by $1(x) = 1$,for all $x \in X$. The universal fuzzy set on a set X denoted by 1_X.

***Definition (1.3)*[2]**

Let A and B be fuzzy sets in a set X. Then we put:

(a) $A = B$ if and only if $A(x) = B(x)$, for all $x \in X$.

(b) $A \le B$ if and only if $A(x) \le B(x)$, for all $x \in X$.

(c) $Z = A \vee B$ if and only if $Z(x) = \max\{A(x), B(x)\}$, for all $x \in X$.

(d) $D = A \wedge B$ if and only if $D(x) = \min\{A(x), B(x)\}$, for all $x \in X$.

(e) $E = A^c$ if and only if

$E(x) = 1 - A(x)$, for all $x \in X$.

***Remark (1.4)* [5]**

Let A and B be fuzzy sets in a set X, then $A \wedge B \leq A$ and $A \wedge B \leq B$.

***Definition (1.5)* [6]**

For any family $\mathcal{A} = \{ A_j : j \in J \}$of fuzzy sets in X, the union, $\bigvee_{j \in J} A_j$ and the intersection, $\bigwedge_{j \in J} A_j$ are defined by:

$\bigvee_{j \in J} A_j(x) = \sup\{A_j(x) : j \in J\}, \ x \in X.$

$\bigwedge_{j \in J} A_j(x) = \inf\{A_j(x) : j \in J\}, \quad x \in X.$

***Definition (1.6)*[1]**

Let X and Y be two non-empty sets, $f: X \longrightarrow Y$ be a mapping. For a fuzzy set B in Y with membership mapping $B(y)$. Then the inverse image of B under f, written as $f^{-1}(B)$, is a fuzzy set in X whose membership mapping is defined by:

$f^{-1}(B)(x) = B(f(x))$ for all $x \in X$. (i.e., $f^{-1}(B) = B \circ f$)

Conversely, let A be a fuzzy set in X with membership mapping$A(x)$. The image of A under f written as $f(A)$, is a fuzzy set in Y whose membership function is given by:

$$f(A)(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z), & if\ f^{-1}(y) \neq \emptyset \\ 0, & if\ f^{-1}(y) = \emptyset \end{cases}$$

for all $y \in Y$, where $f^{-1}(y) = \{x : f(x) = y\}$.

***Definition (1.7)*[4,11]**

A fuzzy point (singleton) in X is a fuzzy set in X defined by:

$$x_r(y) = \begin{cases} r, & if \quad y = x, \\ 0, & otherwise, \end{cases}$$

For each $x \in X$, the single point x is called the support of x_r and $r \in (0,1]$ its value. We denote the class of all fuzzy points in X by $\text{FP}(X)$. A fuzzy point x_r is said to be contained in a fuzzy set A or x_r belongs to A, i.e., $x_r \in A$ if and only if $r \leq A(x)$. Two fuzzy points x_r and y_s in X are said to be distinct if and only if their supports are distinct.

***Definition (1.8)*[9]**

A fuzzy set A in X is called quasi-coincident (in short q-coincident) with a fuzzy set B in X, denoted by AqB if and only if $A(x) + B(x) > 1$, for some $x \in X$. Otherwise if $A(x) + B(x) \leq 1$, for every $x \in X$, then A is called not q-coincident with B and it is denoted by $A\tilde{q}B$.

***Lemma (1.9)* [7]**

Let A, B and C be fuzzy sets in X, $\{A_j : j \in J\}$be a family of fuzzy sets in X, and x_r be a fuzzy point in X. Then:

(a) If $A \wedge B = 0_X$, then $A\tilde{q}B$.
(b) $A\tilde{q}B$ if and only if $A \leq B^c$.

***Proposition (1.10)* [4]**

Let x_r be a fuzzy point in X and A be any fuzzy subset of X. Then $x_r \in A$if and only if$x_r \tilde{q} A^c$.

***Definition (1.11)* [2]**

Let X be a set.A fuzzy topology on X is a family T of fuzzy sets in X, which satisfies the following conditions:

(1)$0_X, 1_X \in T$,

(2) If $A, B \in T$, then $A \wedge B \in T$,

(3) If $A_j \in T$ for each $j \in J$,

then $\bigvee_{j \in J} A_j \in T$.

T is called a fuzzy topology for X, and the pair (X, T) is called a fuzzy topological space (in short fts) and X is called fuzzy space. Every member of T is called a fuzzy open set. A fuzzy set is called fuzzy closed if and only if its complement is fuzzy open.

***Definition (1.12)* [6]**

A fuzzy set A in an fts (X, T) is called a fuzzy quasi-neighborhood (in short q-nhd) of a fuzzy point x_r in X if and only if there exists $B \in T$ such that $x_r qB$ and

$B \leq A$. The family of all fuzzy q-nbds of x_r is called the system of fuzzy q-nbds of x_r and it is denoted by $N^Q_{x_r}$.

Definition (1.13) **[8]**

An fts (X, T) is called fuzzy Hausdorff or fuzzy T_2-space if and only if for any pair of distinct fuzzy points x_r, y_s in X, there exists $A \in N^Q_{x_r}$, $B \in N^Q_{y_s}$ such that $A \wedge B = 0_X$.

Definition (1.14)[7]

Let A be a fuzzy set in X and T be a fuzzy topology on X. Then the induced fuzzy topology on A is the family of fuzzy subsets of A which are the intersection with A of fuzzy open set in X. The induced fuzzy topology is denoted by T_A, and the pair (A, T_A) is called a fuzzy subspace of X.

Definition (1.15) [7]

Let (X, T) be a fuzzy topological space and $\in I^X$. Then :

(i) The union of all fuzzy open sets contained in A is called the fuzzy interior of A

and denoted by A°. i.e. ,

$A^\circ = \vee \{B : B \leq A, B \in T\}$.

(ii) The intersection of all fuzzy closed sets containing A is called the fuzzy closure

of A and denoted by $\overline{A}$. i.e. ,

$$\overline{A} = \wedge \{B : A \leq B, B^c \in T\}.$$

Remarks(1.16) [7]

(i) The interior of a fuzzy set A is the largest open fuzzy set contained in A and trivially , a fuzzy set A is fuzzy open set if and only if $A = A^\circ$.

(ii) The closure of a fuzzy set A is the smallest closed fuzzy set containing A and trivially , a fuzzy set A is fuzzy closed if and only if $A = \overline{A}$.

Proposition (1.17) [12]

Let x_r be a fuzzy point in X and A be any fuzzy set in an fts (X, T), then $x_r \in \overline{A}$ if and only if BqA for every fuzzy open set B in X such that B is q-coincident with x_r.

Definition (1.18) **[2]**

A mapping f from an fts (X, T) into an fts (Y, T) is called fuzzy continuous if and only if the inverse image of each fuzzy open set in Y is a fuzzy open set in X.

Example (1.19)

Let (X, T) be an fts, then the identity mapping $id_X : (X, T) \longrightarrow (X, T)$ is fuzzy continuous.

Proposition (1.20) **[7]**

Let X and Y be fuzzy spaces, and $f : X \longrightarrow Y$ be a mapping, then the following statements are equivalent:

(a) f is fuzzy continuous.

(b) The inverse image of each fuzzy closed set in Y is fuzzy closed in X.

(c) For each fuzzy set B in Y,

$\overline{f^{-1}(B)} \leq f^{-1}(\overline{B})$.

(d) For each fuzzy set A in X,

$f(\overline{A}) \leq \overline{f(A)}$.

(e) For each fuzzy set B in Y,

$f^{-1}(B^\circ) \leq (f^{-1}(B))^\circ$.

Definition (1.22) **[6]**

A mapping f from an fts (X, T) into an fts (Y, T') is called fuzzy closed (fuzzy open) if $f(A)$ is a fuzzy closed (fuzzy open) set in Y for each fuzzy closed set (fuzzy open set) A in X.

***Definition (1.23)* [6]**

A fuzzy set A in an fts X is called fuzzy semi-open if there exists a fuzzy open subset O of X such that $O \leq A \leq \overline{O}$. The complement of a fuzzy semi-open set is defined to be fuzzy semi-closed.

***Proposition (1.24)* [6]**

A fuzzy subset A in an fts X is fuzzy semi-open if and only if $A \leq \overline{A^{\circ}}$.

***Remarks (1.25)* [6]**

(a) Every fuzzy open set is fuzzy semi-open.

(b) Every fuzzy closed set is fuzzy semi-closed.

The converse of a and b of Remark (1.25), are not true in general as the following example.

Example (1.26)

Let A, B, C and D be fuzzy sets of $X = \{a,b,c\}$ defined as follows:

$A = \{a_{0.4}, b_{0.4}, c_{0.3}\}$ $B = \{a_{0.4}, b_{0.3}, c_{0.2}\}$

$C = \{a_{0.4}, b_{0.4}, c_{0.6}\}$ $D = \{a_{0.6}, b_{0.4}, c_{0.6}\}$

Consider the fuzzy topology $T = \{0_X, 1_X, A, B\}$. Notice that C is a fuzzy semi-open set, but it is not fuzzy open. Also D is a fuzzy semi-closed set, but it is not fuzzy closed.

***Definition (1.27)*[4]**

Let A be a fuzzy set of an fts X. Then the fuzzy semi interior of A, denoted by $A^{\circ s}$, is the union of all fuzzy semi open sets in X which are contained in A.It is evident that A is fuzzy semi open if and only if $A^{\circ s} = A$.

***Definition(1.28)* [6]**

Let A be a fuzzy set of an fts X. Then the fuzzy semi-closure of A, denoted by $\overline{A}^{s}$, is the intersection of all fuzzy semi-closed subsets of X containing A. It is clear that A is fuzzy semi-closed if and only if $A = \overline{A}^{s}$.

***Proposition (1.29)* [6]**

For fuzzy sets A and B in an fts X. Then the following hold:

(a) If $A \leq B$, then $\overline{A}^{s} \leq \overline{B}^{s}$.

(b) $\overline{\overline{A}^{s}}^{s} = \overline{A}^{s}$.

(c) $\overline{A \wedge B}^{s} \leq \overline{A}^{s} \wedge \overline{B}^{s}$.

(d) $\overline{A \vee B}^{s} = \overline{A}^{s} \vee \overline{B}^{s}$.

Definition (1.30) [3]

A fuzzy set A in an fts X is called a fuzzy semi quasi-neighborhood of a fuzzy point x_r if there exists a fuzzy semi open set B in X such that $x_r q B \leq A$. The family of all fuzzy semi q-nbds of x_r is called the system of fuzzy semi q-nbds of x_r and it is denoted by $N_{x_r}^{Qs}$.

Remark (1.31)[3]

Every fuzzy q-nbd subset of an fts X is a fuzzy semi q-nbd.

Proposition (1.32)

If A is a fuzzy set in an fts X and $x_r$is a fuzzy point in X, then $x_r \in \overline{A}^{s}$ if and only if BqA for every fuzzy semi open set B in X such that B is q-coincident with x_r.

***Proof* :**

$\Longrightarrow$ Suppose that B is a fuzzy semi open set in X such that $x_r q B$ and $\tilde{q}B$, then $A \leq B^c$. But $x_r \notin B^c$(since $x_r q B$, then $r > B^c(x)$) and B^c is a fuzzy semi closed set in X . Thus $x_r \notin \overline{A}^{s}$.

$\Longleftarrow$ Let $x_r \notin \overline{A}^{s}$, then there exists a fuzzy semi closed set B in X such that $A \leq B$ and $x_r \notin B$, therefore $r > B(x)$, hence $x_r q B^c$ and B^c be a fuzzy semi open

set in X . Since $\leq B$, then by lemma (1.9,b) , $A\tilde{q}B^c$.

***Definition (1.33)* [10]**

A mapping f from an fts X into an fts $Y$is said to be:

(a) fuzzy semi continuous if $f^{-1}(A)$ is a fuzzy semi open set in X, for each fuzzy open set A in Y.

(b) fuzzy semi irresolute if $f^{-1}(A)$ is a fuzzy semi open set in X,for each fuzzy semi open set A in Y.

§ 2. Fuzzy Compact and Fuzzy Semi Compact Space

This section contains the definitions , proportions and theorems about fuzzy compact space and fuzzy semi compact space and we give a new results .

***Definition (2.1)* [6,8]**

A mapping $\mathfrak{F}: D \longrightarrow \mathrm{FP}(X)$ is called a fuzzy net in X and is denoted by $\{\mathcal{S}(n): n \in D\}$, where D is a directed set. If $\mathcal{S}(n) = x_{r_n}^n$ for each $n \in D$, where $x \in X$, $n \in D$ and $r_n \in (0,1]$, then the fuzzy net $\mathfrak{F}$ is denoted as $\{x_{r_n}^n: n \in D\}$ or simply $\{x_{r_n}^n\}$.

***Definition (2.2)* [6,8]**

A fuzzy net $\mathfrak{L} =\{y_{r_m}^m: m \in E\}$ in X is called a fuzzy subnet of a fuzzy net $\mathfrak{F} = \{ x_{r_n}^n: n \in D \}$ if and only if there exists a mapping $f: E \longrightarrow D$ such that :

1. $\mathfrak{L} = \mathfrak{F} \circ f$, that is, $\mathfrak{L}_i = \mathfrak{F}_{f(i)}$ for each $i \in E$.

2. For each $n \in D$ there exists some $m \in E$ such that, if $p \in E$ with $p \geq m$, then $f(p) \geq n$.

We shall denote a fuzzy subnet of a fuzzy net $\{x_{r_n}^n: n \in D\}$ by $\{x_{r_{f(m)}}^{f(m)}: m \in E\}$.

***Definition (2.3)*[6]**

Let (X,T) be an fts and $\mathfrak{F} =\{x_{r_n}^n: n \in D\}$ be a fuzzy net in X and A be a fuzzy set in X. Then $\mathfrak{F}$ is said to be:

(a) Eventually with A if and only if $\exists\, m \in D$, such that $x_{r_n}^n qA$, $\forall\, n \geq m$.

(b) Frequently with A if and only if $\forall\, n \in D$, $\exists\, m \in D$, with $m \geq n$, such that $x_{r_m}^m qA$.

***Definition (2.4)* [8]**

Let (X,T) be an fts, $x_r \in \mathrm{FP}\,(X)$ and let $\mathfrak{F} = \{x_{r_n}^n: n \in D\}$ be a fuzzy net in X. Then:

$(i)x_r$ is called a cluster point of a fuzzy net $\mathfrak{F}$, denoted by $\mathfrak{F} \propto x_r$, if for each $A \in N_{x_r}^Q$, $\mathfrak{F}$ is frequently with A.

$(ii)\mathfrak{F}$ is said to be convergent to x_r and x_r is called a limit point of $\mathfrak{F}$, denoted by $\mathfrak{F} \rightarrow x_r$, if for each $A \in N_{x_r}^Q$, $\mathfrak{F}$ is eventually with A.

Example (2.5)

Let $X =\{a\}$, $A(a) = \frac{2}{3}$ be a fuzzy set in X, and $T =\{0_X, 1_X, A(a)\}$ be a fuzzy topology on X . Define a fuzzy net $\mathfrak{F}: (N, \geq) \longrightarrow \mathrm{FP}(X)$ as follows:

$\mathfrak{F} =\{ a_{\frac{1}{2}}^1, a_{\frac{1}{3}}^2, a_{\frac{1}{2}}^3, a_{\frac{1}{3}}^4, \ldots\ldots\}$, then $\mathfrak{F} \propto a_{\frac{1}{2}}$, because $a_{\frac{1}{2}}^n qA$, where $n =1,\ 3,5,\ldots.$ But $a_{\frac{1}{3}}^n \tilde{q}A$, where $n =2,4,6,\ldots.$ Thus $\mathfrak{F}$ is not convergent to $a_{\frac{1}{2}}$.

Proposition (2.6) [8]

A fuzzy point x_r is cluster point of a fuzzy net $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$ if and only if it has a fuzzy subnet which is convergent to x_r.

Proposition (2.7) [5]

If $x_r \in \mathrm{FP}(X)$ and A be a fuzzy set in X,then $x_r \in \overline{A}$ if and only if there exists a fuzzy net $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$in A, such that it's convergent to x_r.

Proposition (2.8) [5]

An fts X is fuzzy T_2-space if and only if every converges fuzzy net $\mathfrak{F}$ on X has a unique limit point.

Definition (2.9) **[10]**

Let (X, T) be an fts, $x_r \in$ FP (X) and let $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$ be a fuzzy net in X. Then:

$(i)x_r$ is called a semi cluster point of a fuzzy net $\mathfrak{F}$, denoted by $\mathfrak{F} \propto^s x_r$, if for each $A \in N_{x_r}^{Qs}$, $\mathfrak{F}$ is frequently with A.

$(ii)\mathfrak{F}$ is said to be semi convergent to x_r and x_r is called a semi limit point of $\mathfrak{F}$, denoted by $\mathfrak{F} \xrightarrow{s} x_r$, if for each $A \in N_{x_r}^{Qs}$, $\mathfrak{F}$ is eventually with A.

Proposition (2.10) **[10]**

A fuzzy point x_r issemi cluster point of a fuzzy net $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$ if and only if it has a fuzzy subnet which is semi convergent to x_r.

Proposition (2.11)

If$x_r \in \mathrm{FP}(X)$ and A be a fuzzy set in X,then $x_r \in \overline{A}^s$ if and only if there exists a fuzzy net $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$in A, such that it's semi convergent to x_r.

Proof **:**

$\Longrightarrow$ Let $x_r \in \overline{A}^s$, then BqA for each fuzzy semi open set B in X such that $x_r qB$. That is, there exists $r_B \in (0,1]$, such that $x_{r_B}^B \in A$ and $x_{r_B}^B qB$. Let $D = N_{x_r}^{Qs}$. Then $(D, \geqslant)$ is a directed set under the inclusion relation so we can define a fuzzy net $\mathfrak{F}: N_{x_r}^{Qs} \longrightarrow$ FP (X) given by $\mathfrak{F} = x_{r_B}^B$, $\forall\, B \in N_{x_r}^{Qs}$. Then $\mathfrak{F}$ is a fuzzy net in A. Now, let $P \in N_{x_r}^{Qs}$, such that $P \geqslant B$ iff $P \leq B$, so there exists a fuzzy net $\{x_{r_P}^P\}_{P \in N_{x_r}^{Qf}}$ such that $x_{r_P}^P qP$. Then $x_{r_P}^P qB$. So $\mathfrak{F} \xrightarrow{s} x_r$.

$\Longleftarrow$ Let $\mathfrak{F}$ be a fuzzy net in A, such that $\mathfrak{F} \xrightarrow{s} x_r$, and let B be a fuzzy semi open set in X , such that $x_r qB$, then $\forall\, n \in D$, $\exists\, m \in D$ such that $x_{r_n}^n qB$ with $n \geq m$ and so BqA for each fuzzy semi open set B in X, such that $x_r qB$. Thus by Proposition (1.32), $x_r \in \overline{A}^s$.

Corollary (2.12)

If $x_r \in \mathrm{FP}(X)$ and A be a fuzzy set in X,then $x_r \in \overline{A}^s$ if and only if there exists a fuzzy net $\mathfrak{F} = \{x_{r_n}^n : n \in D\}$in A, such that $\mathfrak{F} \propto^s x_r$.

Proof **:**

$\Longrightarrow$By proposition (2.11).

$\Longleftarrow$Let $\mathfrak{F}$ be a fuzzy net in A, such that $\mathfrak{F} \propto^s x_r$, and let B be a fuzzy semi open set in X, such that $x_r qB$, then $\forall\, m \in D$, $\exists\, n \in D$ such that $x_{r_n}^n qB$ with $n \geq m$ and so BqA for each fuzzy semi open set B in X, such that $x_r qB$. Thus by proposition (1.32), $x_r \in \overline{A}^s$.

Definition (2.13)

An fts (X, T) is called fuzzy semi Hausdorff or fuzzy semi T_2- space if and only if for any pair of distinct fuzzy points x_r, y_s in X, there exists $A \in N_{x_r}^{Qs}$, $B \in N_{y_s}^{Qs}$ such that $A \wedge B = 0_X$.

Proposition (2.14)

An fts X is fuzzy semi T_2-space if and only if every semi converges fuzzy net $\mathfrak{F}$ on X has a unique semi limit point.

Proof **:**

$\Longrightarrow$ Let $\mathfrak{F}$ be a fuzzy net in X, such that $\mathfrak{F} \xrightarrow{s} x_r$ and $\mathfrak{F} \xrightarrow{s} y_s$, such that $x \neq y$. Since $\mathfrak{F} \xrightarrow{s} x_r$, we have for each $A \in N_{x_r}^{Qs}$, there exists $m \in D$, such that $x_{r_n}^n qA$, $\forall\, n \geq m$.

Also, since $\mathfrak{F} \overset{s}{\to} y_s$, we have for each $B \in N_{y_s}^{Qs}$, there exists $k \in D$, such that $x_{r_n}^n qB, \forall\, n \geq k$. Since $(D, \geqslant)$ is a directed set, then there exists $p \in D$, such that $p \geq m$ and $p \geq k$, then $x_{r_n}^n qA$, $\forall n \geq p$ and $x_{r_n}^n qB, \forall\, n \geq p$, therefore $A \wedge B \neq 0_X$ for each $A \in N_{x_r}^{Qs}$ and for each $B \in N_{y_s}^{Qf}$, thus X is not a fuzzy semi T_2-space.

$\Leftarrow$ Let X be not a fuzzy semi T_2-space, then there exists $x_r, y_s \in \mathrm{FP}(X)$, such that $x \neq y$ and $A \wedge B \neq 0_X$, $\forall A \in N_{x_r}^{Qs}$ and $\forall B \in N_{y_s}^{Qs}$.
Put $N_{x_r, y_s}^{Qs} = \{ A \wedge B : A \in N_{x_r}^{Qs}$ and $B \in N_{y_s}^{Qs} \}$.
Thus $\forall F \in N_{x_r, y_s}^{Qs}$, $\exists\, x_{r_F}^F qF$. Then $\{x_{r_F}^F\}_{F \in N_{x_r, y_s}^{Qs}}$ is a fuzzy net in X.To prove $x_{r_F}^F \overset{s}{\to} x_r$ and $x_{r_F}^F \overset{s}{\to} y_s$. Let $E \in N_{x_r}^{Qs}$, then $E \in N_{x_r, y_s}^{Qs}$ $(E = E \wedge 1_X)$. Thus $x_{r_F}^F qE$, $\forall\, F \leq E$, hence $x_{r_F}^F \overset{s}{\to} x_r$. Also $x_{r_F}^F \overset{s}{\to} y_s$, so $\{x_{r_F}^F\}_{F \in N_{x_r, y_s}^{Q}}$ has two semi limit points.

Definition* (*2.15) [2]

A family Ω of fuzzy sets is a cover of a fuzzy set A if and only if $A \leq \vee\{G_i : G_i \in \Omega\}$ and it is called a fuzzy open cover if and only if Ω is a cover of A and each member of Ω is a fuzzy open set. A subcover of Ω is a subfamily of Ω which is also a cover of A.

Definition* (*2.16) [6]

Let A be a fuzzy set in an fts X. Then A is said to be fuzzy compact if for every fuzzy open cover of A has a finite subcover of A .Also, an fts X is called fuzzy compact if every fuzzy open cover of X has a finite sub cover.

Example (2.17)

The indiscrete fuzzy topological space is fuzzy compact.

***Proposition* (*2.18*) [5]**

The fuzzy continuous image of a fuzzy compact set is fuzzy compact.

Proposition* (*2.19) [7]

Let Y be a fuzzy subspace of an fts X, and A be a fuzzy set in Y. Then A is a fuzzy compact set in X if and only if A is a fuzzy compact set in Y.

Proposition* (*2.20) [2]

An fts is fuzzy compact if and only if each family of fuzzy closed sets which has the finite intersection property has a non-empty intersection.

Corollary* (*2.21) [7]

A fuzzy closed subset of a fuzzy compact space is fuzzy compact.

Definition*(*2.22) [7]

A fuzzy filter base on X is a nonempty subset $\mathcal{F}$ of I^X such that

$(i) 0_X \in \mathcal{F}$.

(ii) If $A_1, A_2 \in \mathcal{F}$, then $\exists\, A_3 \in \mathcal{F}$ such that $A_3 \leq A_1 \wedge A_2$.

Definition*(*2.23) [10]

A fuzzy point $x_\propto$ in a fuzzy topological space X is said to be a fuzzy semi- cluster point of a fuzzy filter base $\mathcal{F}$ on X if $x_\propto \leq \overline{B}$, for all $\in \mathcal{F}$.

Proposition* (*2.24) [7]

A fuzzy topological space (X, T) is fuzzy compact if and only if every fuzzy filter base on X has a fuzzy cluster point .

Proposition* (*2.25) [7]

An fts X is fuzzy compact if and only if every fuzzy net in X has a $cluster$ point.

Proposition (2.26) [7]

An fts X is fuzzy compact if and only if each fuzzy net in X has a Q-convergent fuzzy subnet.

Proposition (2.27) [7]

A fuzzy compact subset of a fuzzy T_2-space is fuzzy closed.

Proposition (2.28) [7]

In any fts X, the intersection of any fuzzy closed set with any fuzzy compact set is fuzzy compact.

Definition (2.29)[7]

Let X be an fts. A fuzzy subset V of X is called fuzzy compactly closed if for every fuzzy compact set K in X, $V \wedge K$ is fuzzy compact.

Example (2.30)

Every fuzzy subset of a fuzzy indiscrete space is fuzzy compactly closed.

Proposition (2.31) [7]

Let X be a fuzzy T_2-space. A fuzzy subset A of X is fuzzy compactly closed if and only if A is fuzzy closed.

Definition (2.32) **[10]**

A family Ω of fuzzy sets is called a fuzzy semi open cover if Ω is a cover of A and each member of Ω is a fuzzy semi open set. A subcover of Ω is a subfamily of Ω which is also a cover of A.

Definition (2.33) **[10]**

Let A be a fuzzy set in an fts X. Then A is said to be fuzzy semi compact if for every fuzzy semi open cover of A has a finite subcover of A.Also, an fts X is called fuzzy semi compact if every fuzzy semi open cover of X has a finite sub cover.

Remarks(2.34)

Every fuzzy semi compact space is fuzzy compact.

Proposition (2.35)

An fts X is fuzzy semi compact if and only if each family of fuzzy semi closed sets which has the finite intersection property has a non-empty intersection.

***Proof* :**

$\Rightarrow$Let $\{A_j : j \in J\}$ be a family of fuzzy semi closed sets with the finite intersection property. Supposed that $\bigwedge_{j\in J} A_j = 0_X$. Then $\bigvee_{j\in J} A^c{}_j = 1_X$. Since $\{A^c{}_j : j \in J\}$ is a collection of fuzzy semi open sets cover of X, then from the semi-compactness of X it follows that there exists a finite subset $F \subseteq J$ such that $\bigvee_{j\in F} A^c{}_j = 1_X$. Then $\bigwedge_{j\in F} A_j = 0_X$ which gives a contradiction and therefore $\bigwedge_{j\in J} A_j \neq 0_X$.

$\Leftarrow$Let $\{A_j : j \in J\}$ be a collection of fuzzy semi open sets cover of X. Suppose that for every finite subset $F \subseteq J$, we have $\bigvee_{j\in F} A_j \neq 1_X$. Then $\bigwedge_{j\in F} A^c{}_j \neq 0_X$. Hence $\{A^c{}_j : j \in J\}$ satisfies the finite intersection property. Then from hypothesis we have $\bigwedge_{j\in J} A^c{}_j \neq 0_X$ which implies $\bigvee_{j\in J} A_j \neq 1_X$ and this contradicting that $\{A_j : j \in J\}$ is a fuzzy semi open cover of X. Then X is fuzzy semi compact.

Proposition (2.36) [10]

A fuzzy topological space (X, T) is fuzzy semi compact if and only if every fuzzy filter base on X has a fuzzy semi-cluster point .

Proposition (2.37) [10]

An fts X is fuzzy semi compact if and only if every fuzzy net in X has a fuzzy semi-cluster point.

Proposition (2.38) [10]

An fts X is fuzzy semi compact if and only if each fuzzy net in X has semi convergent fuzzy subnet.

Proposition (2.39)

In any fts X, the intersection of any fuzzy semi closed set with any fuzzy semi compact set is fuzzy semi compact.

***Proof* :**

Let A be a fuzzy semi closed set and B be a fuzzy semi compact set in X. Suppose that $\{x_{r_n}^n\}_{n\in D}$ be a fuzzy net in $A\wedge B$, then by remark (1.4), $\{x_{r_n}^n\}_{n\in D}$ in A and B. Since B is fuzzy semi compact, then by proposition (2.38), $\{x_{r_n}^n\}_{n\in D}$ has a semi convergent fuzzy subnet. Since A is a fuzzy semi closed set in X, then $x_r \in \overline{A}^s = A$, therefore $\{x_{r_n}^n\}_{n\in D}$ be a fuzzy net in $A\wedge B$ which has a semi convergent fuzzy subnet, hence by proposition (2.38), $A\wedge B$ is fuzzy semi compact.

Corollary (2.40)

A fuzzy semi closed subset of a fuzzy semi compact space is fuzzy semi compact.

***Proof* :**

Let A be a fuzzy semi closed set of a fuzzy semi compact space (X,T). Since $A\wedge 1_X = A$. Then by proposition (2.39), A is a fuzzy semi compact set in X.

Proposition (2.41)

A fuzzy semi compact subset of a fuzzy semi T_2-space is fuzzy semi closed.

***Proof* :**

Let A be a fuzzy semi compact subset of a fuzzy semi T_2-space. We must show that A is a fuzzy semi closed set. If $x_r \in \overline{A}^s$, then by proposition (2.11), there exists a fuzzy net $\{x_{r_n}^n\}_{n\in D}$ in A such that $x_{r_n}^n \xrightarrow{s} x_r$. But A is fuzzy semi compact, then by theorem (2.37), $\{x_{r_n}^n\}_{n\in D}$ has a semi cluster point y_s in A and thus there exists a fuzzy subnet in A which is semi converges to y_s. Since X is a fuzzy semi T_2 -space, then by proposition (2.14), $x_r = y_s$. Thus $x_r \in A$, showing that A is fuzzy semi closed.

Lemma (2.42)

Let X be an fts and A be a fuzzy semi closed subset of X. Then $A\wedge B$ is fuzzy semi compact set in A, for every fuzzy semi compact set B in X.

***Proof* :**

Let B be a fuzzy semi compact set in X. To prove $A\wedge B$ is a fuzzy semi compact set in A. Let $\{x_{r_n}^n\}_{n\in D}$ be a fuzzy net in $A\wedge B$. Since A is a fuzzy semi closed set in X, then by proposition (2.39), $A\wedge B$ is a fuzzy semi compact set in X. Thus by theorem (2.37), the fuzzy net $\{x_{r_n}^n\}_{n\in D}$ has a semi cluster point x_r in $A\wedge B$, therefore by corollary (2.12) and proposition (1.29,c), $x_r \in A$. Thus by theorem (2.37), $A\wedge B$ is a fuzzy semi compact set in A.

Proposition (2.43)

Let $f:(X,T) \longrightarrow (Y,T')$ be a mapping. Then:

(a) If f is fuzzy semi continuous, then an image $f(A)$ of any fuzzy semi compact set A in X is a fuzzy compact subset of Y.

(b) If f is fuzzy semi irresolute, then an image $f(A)$ of any fuzzy semi compact set A in X is a fuzzy semi compact subset of Y.

***Proof* :**

(a) Since every fuzzy semi compact set is fuzzy compact, then A is a fuzzy compact set in X and since f is a fuzzy continuous mapping, thus by proposition (2.18), $f(A)$ is a fuzzy compact set in Y.

(b) Let $\Omega = \{ G_j : j \in J \}$ be a family of fuzzy semi open set cover of $f(A)$. Since f is fuzzy semi irresolute, then $\{ f^{-1}(G_j) : j \in J \}$ is a fuzzy semi open cover of A. Since A is a fuzzy semi compact set in X, there is a finite subfamily $\{f^{-1}(G_j) : j = 1,2,\ldots, n\}$, such that $A \leq \vee_{j\in J}^{n} f^{-1}(G_j)$ which implies

$A \leq f^{-1}(\vee_{j\in J}^{n} G_j)$ and then $f(A) \leq f(f^{-1}(\vee_{j\in J}^{n} G_j)) \leq \vee_{j\in J}^{n} G_j$. Therefore $f(A)$ is a fuzzy semi compact set in Y.

Proposition (*2.44*)

Let X be a fuzzy semi compact and fuzzy T_2-space and A be a fuzzy set in X. Then:

(a) A is fuzzy closed if and only if A is fuzzy semi closed.

(b) A is fuzzy compact if and only if A is fuzzy semi compact.

***Proof* :**

(a) $\Longrightarrow$ By remark (1.25,b).
$\Longleftarrow$ Let A be a fuzzy semi closed set in X. Since X is a fuzzy semi compact space, then by corollary (2.40), A is a fuzzy semi compact set, so it is a fuzzy compact set. Now, let K be a fuzzy compact set in X, then $A \wedge K$ is fuzzy compact, thus A is a fuzzy compactly closed set in X. Since X is a fuzzy T_2 -space, then by theorem (2.31), A is a fuzzy closed set.

(b) $\Longrightarrow$ Let A be a fuzzy compact set in X. Since X is a fuzzy T_2 -space, then by proposition (2.27), A is a fuzzy closed set in X, and then it is a fuzzy semi closed set in X. Since X is a fuzzy semi compact space, then by corollary (2.40), A is a fuzzy semi compact set in X.
$\Longleftarrow$ By proposition (2.34).

§ 3. Fuzzy Compact Mappings And Fuzzy Semi Compact Mappings

The section will contain the concept of fuzzy compact mapping and fuzzy semi compact mapping and we give new results.

Definition (*3.1*) [7]

A mapping f from an fts X into an fts Y is called fuzzy compact if the inverse image of each fuzzy compact set in Y, is a fuzzy compact set in X.

Example (*3.2*)

Every mapping from a finite fts into any fts is fuzzy compact.

Proposition (*3.3*) [6]

Let X, Y and Z be fuzzy spaces, and $f: X \longrightarrow Y$, $g: Y \longrightarrow Z$ be mappings. Then:

(a) If f is fuzzy compact and g is fuzzy compact, then $g \circ f$ is a fuzzy compact mapping.

(b) If $g \circ f$ is fuzzy compact, f is onto and fuzzy continuous, then g is fuzzy compact.

(c) If $g \circ f$ is fuzzy compact, g is one to one and fuzzy irresolute, then f is fuzzy compact

Definition (*3.4*)

A mapping f from an fts X into an fts Y is called fuzzy semi compact if the inverse image of each fuzzy semi compact set in Y, is a fuzzy compact set in X.

Remark (*3.5*)

Every fuzzy compact mapping is fuzzy semi compact.

Proposition (*3.6*)

Let X, Y and Z be fuzzy spaces, and $f: X \longrightarrow Y$, $g: Y \longrightarrow Z$ be mappings. Then:

(a) If f is fuzzy compact and g is fuzzy semi compact, then $g \circ f$ is a fuzzy semi compact mapping.

(b) If $g \circ f$ is fuzzy semi compact, f is onto and fuzzy continuous, then g is fuzzy semi compact.

(c) If $g \circ f$ is fuzzy semi compact, g is one to one and fuzzy semi irresolute, then f is fuzzy semi compact.

***Proof* :**

(a) Let A be a fuzzy semi compact set in Z, then $g^{-1}(A)$ is a fuzzy compact set in Y, and so $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a fuzzy compact set in$X$.Hence $g \circ f: X \longrightarrow Z$ is a fuzzy semi compact mapping.

(b) Let A be a fuzzy semi compact set in Z, then $(g \circ f)^{-1}(A)$ is a fuzzy compact set in X and then $(f(g \circ f)^{-1})(A)$ is a fuzzy compact set in Y. Now, since f is onto, then $(f(g \circ f)^{-1})(A) = g^{-1}(A)$,

hence $g^{-1}(A)$ is a fuzzy compact set in Y. Therefore g is a fuzzy semi compact mapping.

(c) Let A be a fuzzy semi compact set in Y, then by proposition (2.43,b), $g(A)$ is a fuzzy semi compact set in Z, thus $(g\circ f)^{-1}(g(A))$ is a fuzzy compact set in X. Since g is one to one, then $(g\circ f)^{-1}\big(g(A)\big)=f^{-1}(A)$, hence $f^{-1}(A)$ is a fuzzy compact set in X. Thus f is a fuzzy semi compact mapping.

§.4 Fuzzy Coercive And Fuzzy Semi Coercive Mapping .

The section will contain the definition of a fuzzy coercive and fuzzy semi coercive mapping and the relation between fuzzy semi compact mapping and the fuzzy semi coercive mapping .

Definition (4.1) **[7]**

Let X and Y be fuzzy spaces. A mapping $f: X \longrightarrow Y$ is called fuzzy coercive if for every fuzzy compact set B in Y, there exists a fuzzy compact set A in X such that $f(1_X\backslash A) \leq (1_Y\backslash B)$.

Example (4.2)

If X is a fuzzy compact space, then the mapping $f: X \longrightarrow Y$ is fuzzy coercive.

Proposition (4.3) **[7]**

Every fuzzy compact mapping is fuzzy coercive.

Proposition (4.4) **[7]**

Let X and Y be fuzzy spaces, such that Y is a fuzzy T_2 -space. If $f: X \longrightarrow Y$ is fuzzy continuous. Then f is fuzzy compact if and only if it is fuzzy coercive.

Definition (4.5)

A mapping f from an fts X into an fts Y is called fuzzy semi coercive if for every fuzzy semi compact set B in Y, there exists a fuzzy compact set A in X such that $f(1_X\backslash A) \leq (1_Y\backslash B)$.

Example (4.6)

If X is a fuzzy compact space, then the mapping $f: X \longrightarrow Y$ is fuzzy semi coercive. Let B be a fuzzy semi comp4act set in Y. Since X is fuzzy compact and $f(1_X\backslash 1_X)=f(0_X)=0_Y\leq(1_Y\backslash B)$, then f is a fuzzy semi coercive mapping.

Proposition (4.7)

If $f: X \longrightarrow Y$ is a fuzzy coercive mapping and $g: Y \longrightarrow Z$ is a fuzzy semi coercive mapping, then $g\circ f: X \longrightarrow Z$ is a fuzzy semi coercive mapping.

Proof **:**

Let C be a fuzzy semi compact set in Z . Since g is a fuzzy semi coercive mapping, then there exists a fuzzy compact set B in Y, such that $g(1_Y\backslash B)\leq 1_Z\backslash C$. Since f is a fuzzy coercive mapping, then there exists a fuzzy compact set A in X, such that $f(1_X\backslash A)\leq(1_Y\backslash B)$, then $g\big(f(1_X\backslash A)\big)\leq g(1_Y\backslash B)\leq 1_Z\backslash C$, hence $(g\circ f)(1_X\backslash A)\leq 1_Z\backslash C$. Thus $g\circ f$ is a fuzzy semi coercive mapping.

Proposition (4.8)

Every fuzzy coercive mapping is fuzzy semi coercive.

Proof **:**

Let $f: X \longrightarrow Y$ be a fuzzy coercive mapping, and B be a fuzzy semi compact set in Y, so it is a fuzzy compact set, since f is fuzzy coercive, then there exists a fuzzy compact set A in X , such that $f(1_X\backslash A)\leq 1_Y\backslash B$. Hence f is a fuzzy semi coercive mapping.

Proposition (4.9)

If Y is a fuzzy semi compact and fuzzy T_2 -space,then $f: X \longrightarrow Y$ is a fuzzy coercive mapping if and only if it is fuzzy semi coercive.

Proof **:**

$\Longrightarrow$ By proposition (4.8).

$\Longleftarrow$ Let B be a fuzzy compact set in Y. Since Y is a fuzzy semi compact and fuzzy T_2 -space, then by proposition (2.44,b), B is a fuzzy semi compact set in Y , since f is a fuzzy semi coercive mapping, then there exists a fuzzy compact set A in X, such that $f(1_X \backslash A) \leq 1_Y \backslash B$. Hence f is a fuzzy coercive mapping.

Proposition (*4.10*)

Every fuzzy semi compact mapping is fuzzy semi coercive.

Proof **:**

Let $f\colon X \longrightarrow Y$ be a fuzzy semi compact mapping. To prove that f is a fuzzy semi coercive mapping. Let A be a fuzzy semi compact set in Y. Since f is fuzzy semi compact, then $f^{-1}(A)$ is a fuzzy compact set in X.Therefore $f(1_X \backslash f^{-1}(A)) \leq 1_Y \backslash A$. Hence $f\colon X \longrightarrow Y$ is a fuzzy semi coercive mapping.

Proposition (*4.11*)

If Y is a fuzzy T_2-space, and $f\colon X \longrightarrow Y$ be a fuzzy continuous mapping,then f is fuzzy semi compact if and only if f is fuzzy semi coercive.

Proof **:**

$\Longrightarrow$ By proposition (4.10).

$\Longleftarrow$ Let B be a fuzzy semi compact set in Y, so it is fuzzy compact. To prove that $f^{-1}(B)$ is a fuzzy compact set in X. Since Y is a fuzzy T_2-space and f is a fuzzy continuous mapping, then by proposition (2.27) and proposition (1.20), $f^{-1}(B)$ is a fuzzy closed set in X. Since f is a fuzzy semi coercive mapping, then there exists a fuzzy compact set A in X , such that $f(1_X \backslash A) \leq 1_Y \backslash B$. Then $f(A^c) \leq B^c$, therefore $f^{-1}(B) \leq A$, then by corollary (2.21), $f^{-1}(B)$ is a fuzzy compact set in X . Hence f is a fuzzy semi compact mapping.

Proposition (*4.12*)

If Y is a fuzzy semi compact space and fuzzy T_2- space, then for a fuzzy space X and a fuzzy continuous mapping $: X \longrightarrow Y$, the following statements are equivalent:

(a) f is a fuzzy coercive mapping.

(b) f is a fuzzy compact mapping.

(c) f is a fuzzy semi compact mapping.

(d) f is a fuzzy semi coercive mapping.

Proof **:**

(a)$\Longrightarrow$(b) By proposition (4.4).

(b)$\Longrightarrow$(c) By remark (3. 5).

(c)$\Longrightarrow$(d) By proposition (4.10).

(d)$\Longrightarrow$(a) By proposition (4.9)

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الخلاصة

قدمنا في هذا البحث مفهومين التطبيق المتراص شبه الضبابي والتطبيق الاضطراري شبه الضبابي في الفضاء التبولوجي الضبابي. وناقشنا العديد من الصفات والخصائص المهمة. كذلك وضحنا العلاقة بين التطبيق المتراص شبه الضبابي والتطبيق الاضطراري شبه الضبابي .