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Completely Semi Prime Ideal With Respect To An Element Of A Near Ring

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Abstract

In this paper ,we introduce the notions of completely semi prime ideal with respect to an element x (x-C.S.P.I) of a near ring and the completely semi prime ideal near ring with respect to an element x (x-C.S.P.I) . 1. The image and inverse image of x-C.S.P.I under epimomorphism and the 2. direct product of x-C.S.P.I near ring are studied .

Introduction

N.J.Groenewald have show several commutability theorems for completely semi prime ideal of near ring [4] .

Key Words

Near ring ,ideal of near ring , Boolean near ring,near-ring homomorphism, the direct product of the near-rings, completely semi prime ideal, Factor near ring,

1. **Preliminaries**

In this section we give some basic concepts that we need in the second section.

Definition (1.1) [2]

A left near ring is a set N together with two binary operations “+” and ”.” such that

a. (N,+) is a group (not necessarily abelian)

b. (N, .) is a semigroup.

c. (n1 + n2) . n3 = n1 . n3 + n2 . n3

For all n1, n2, n3, ∈ N;

Definition (1.2) [3]:

Let N be a near-ring. A normal subgroup I of (N,+) is called a left ideal of N if

i. IN ⊆ I.

ii. $\forall$ ***n,*** $n_1 \in N$ ***and for all*** $i \in I$ ***,***

$n.(n_1 + i) - n.n_1 \in I$.

Remark (1.3)

We will refer that all near rings and ideals in this paper are left .

Definition (1.4) ([6] page 30)

A near ring N is Boolean if and only if for all $x \in N, x^2 = x$

Definition (1.5)([6]: page 22)

Let $\{Nj\}_{j \in J}$ **be a family of near rings , J is an index set and**

$\prod_{j \in J} N_j = \{(x_j) : x_j \in N_j, \textit{for} \text{ all } j \in J\}$ **be the directed product of** N_j **with the component wise defined operations '+' and '.', is**

called the direct product near ring of the near rings N_j

Definition (1.6) [1]

If I_1 **and** I_2 **are ideals of a near ring N then** $I_1 \cdot I_2 = \{i_1 \cdot i_2 : i_1 \in I_1, i_2 \in I_2\}$.

Definition (1.7) ([6] page 20)

A near ring *N is called an integral domain if N has non -zero divisors*

Definition (1.8)([6] page 21)

Let N_1 **and** N_2 **be two near-rings. The mapping**

$f : N_1 \to N_2$ **is called a near-ring homomorphism if for all m, n** $\in N_1$

f(m + n) = f(m) + f(n) and f(m. n) = f(m) f(n).

Theorem (1.9) ([6] page 23)

Let $f : N_1 \to N_2$ **is homomorphism**

(1) If I is ideal of a near ring N_1 then f(I) is ideal of a near ring N_2.

(2) If J is ideal of a near ring N_2then $f^{-1}(J)$ is ideal of a near ring N_1.

Definition (1.10) [4]

An ideal I of N is called completely semi prime ideal(C.S.P.I) of a near ring .if $x^2 \in I$ implies $x \in I$ for any $x \in N$.

Remark ([6] page 20)

The Factor near ring N/I **is defined as in case of rings .**

Definition (1.11)[5]

Let I be an ideal of a near ring N. Then I its called completely prime ideal of N if $\forall x, y \in N, x.y \in I$ implies $x \in I$ or $y \in I$ **, denoted by C.P.I of N .**

2. The main Results

In this section we study completely semi prime ideal with respect to the element x and completely semi prime ideal near ring with respect to the element x .

Definition (2.1)

let N be a near ring and $x \in N$**, I is called completely semi prime ideal with respect to the element x denoted by (x-C.S.P.I) or (x- completely semi prime ideal)of N if for all** $y \in N$, if $x \cdot y^2 \in I$ implies $y \in I$ **.**

Example (2.2)

Considers N ={0,a,b,c} be a near ring with addition and multiplication defined by the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

I **={0,a} is an ideal of N .**

Then

- I is b-completely semi prime ideal of the near ring N .
- I is not 0-completely semi prime ideal of the near ring N since $0 \cdot b^2 \in I$ *but* $b \notin I$.
- I is not a-completely semi prime ideal in near ring since $a \cdot b^2 \in I$ *but* $b \notin I$.

Proposition (2.3)

Let $\{I_j\}_{j \in J}$ **be a family of x-C.S.P.I of a near ring N for all** $j \in J$ **,** $x \in N$ **.Then** $\bigcap_{j \in J} I_j$ **is a x-C.S.P.I**

Proof

Let $y \in N$ **such that** $x \cdot y^2 \in \bigcap_{j \in J} I_j$,this implies $x \cdot y^2 \in I_j , \forall j \in J$

$\Rightarrow y \in I_j , \forall j \in J$ [since each I_j is a x-C.S.P.I $\forall j \in J$]

$\Rightarrow y \in \bigcap_{j \in J} I_j$

$\Rightarrow \bigcap_{j \in J} I_j$ is a x-C.S.P.I of N

Remark

If I_1 and I_2 be two x-C.S.P.I of a near ring N then the ideal $I_1 . I_2$ of N may be not x-C.S.P.I

Example(2.4)

Considers N={ 0,1,2,3} be a near ring with addition and multiplication defined by the following tables

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	0	0	0
3	0	0	2	3

The ideals I_1 ={0,2} and I_2={0,2,3} are 1-completely semi prime ideal of a near ring of N .

***** I_1**.I_2 ={0} is not 1-completely semi prime ideal of the near ring N [Since** $1.2^2 = 1.0 = 0 \in I_1 . I_2$ but $2 \notin I_1 . I_2$ **]**

Proposition (2.5)

Let N be a Boolean near ring and I be (C.P.I) of N .Then I is x-C.S.P.I for all $x \notin I$ and $x \in N$ **.**

Proof

Let $y \in N$ **such that** $x \cdot y^2 \in I$ **.Then** $x.y \in I$ **[since N is Boolean near ring (y ^{2}=y).Then** $y \in I$ **[since I is C .P.I of N and** $x \notin I$ **].this implies I is x-C.S.P.I .**

Proposition (2.6)

let N be a near ring with multiplicative identity e' **then I is** e' **- C.S.P.I of the near ring N if and only if it is a C.S.P.I of N .**

proof

$\Rightarrow$

let I be an e' **- C.S.P.I of N and** $y \in N$ such that $y^2 \in \mathrm{I}$ **,** $y^2 = e' \cdot y^2 \in I$

then $y \in \mathrm{I}$ **[since I is** e'**- C.S.P.I] .This implies I is C.S.P.I.**

$\Leftarrow$

To prove I is e'**- C.S.P.I . Let I be a C.S.P.I of N and** $y \in N$ **such that**

$e' \cdot y^2 \in I$

$e' \cdot y^2 = y^2 \in I$.

This imples $y \in \mathrm{I}$

[since I is C.S.P.I of N].

Then we have I is e'- C.S.P.I of N

Remark

In general not all completely semi prime ideals of a near ring N are x-completely semi prime ideals of a near ring where $x \in N$ as in the following example .

Example (2.7)

Let N ={0,a,b,c} be a near ring with addition and multiplication defined as

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

The ideal I ={0,a} is a completely semi prime ideal of the near ring N , but it is not a-completely semi prime ideal of a near ring N. Since $a \cdot b^2 \in \mathrm{I}$ but $b \notin \mathrm{I}$.

Proposition (2.8)

If N is non zero near ring and I={0} then I is not 0-.C.S.P.I of the near ring N .

Proof

Suppose I is 0 -C.S.P.I of N , since $N \neq \{0\}$.Then there exist

$y \in N$ such that $y \neq 0$.

$0.y^2 = 0 \in I \Rightarrow y \in \mathrm{I}$ [since I is 0-C.S.P.I]

$\Rightarrow y = 0$

and this contradiction

[since $y \neq 0] \Rightarrow I$ is not 0-.C.S.P.I of N .

proposition (2.9)

let I be nontrivial ideal of the near ring N then I is not 0-.C.S.P.I of N .

Proof

Suppose I is 0 - C.S.P.I of N and

$let\ y \in N \Rightarrow 0.y^2 = 0 \in I \Rightarrow y \in I$ [since I is 0- C.S.P.I of N] $\Rightarrow N \subseteq I$

this contradiction [since I $\subset$ N] $\Rightarrow$I is not 0- C.S.P.I of N .

Theorem (2.10)

Let N_1 and N_2 be two near ring ,

$f : N_1 \to N_2$ be epimomorphism and I be x- C.S.P.I of N_1 such that $\ker f \subseteq I$.

Then f(I) is f(x)- C.S.P.I of N_2 .

Proof

Let I be x- C.S.P.I of N_1 $f(I) = \{f(i) : i \in I\}$

is an ideal of N_2 .To proof f(I) is a f(x)- C.S.P.I of N_2 .

Let $c \in N_2$ such that

$f(x).c^2 \in f(I)$
$\Rightarrow f(x).c^2 = f(x).(f(y))^2$
$= f(x).f(y^2) = f(x.y^2) \in f(I)$
$, where\ c = f(y), y \in N_1$

[since f is an epimomorphism]

$\Rightarrow x.y^2 \in I \Rightarrow y \in I$ [since I is x-C.S.P.I of N_1]

$\Rightarrow c = f(y) \in f(I) \Rightarrow f(I)$ is a f(x)-C.S.P.I of N_2

theorem (2.11)

Let N_1 and N_2 be two near ring .

$f : N_1 \to N_2$ be epiomomorphism and J be a y- C.S.P.I of N_2 . Then f^{-1} (J) is a x- C.S.P.I of N_1 where y= f(x) .

Proof

$f^{-1}(J) = \{x \in N_1 : f(x) \in J\}$ is an ideal of the near ring N_1

Let $z \in N_1$ such that
$x.z^2 \in f^{-1}(J)$
$\Rightarrow f(x.z^2) \in J$

$f(x \cdot z^2) = f(x).f(z^2)$
$= f(x).(f(z))^2$
$= y.(f(z))^2 \in J$
$\Rightarrow f(z) \in J$
[since J *is* y - C.S.P.I of N_2]
$\Rightarrow z \in f^{-1}(J)$
$\to f^{-1}(J)$ is x - C.S.P.I of N_1

Proposition (2.12)

If N is a near integral domain then {0} is x-C.P.S.I for all $x \in N \setminus \{0\}$.

Proof

Let $y \in N$, $x \cdot y^2 \in \{0\} \Rightarrow x \cdot y^2 = 0$ .This implies $y^2 = 0 \Rightarrow y = 0$ then we have

$y \in \{0\}$ [since N is integral demean and $x \neq 0$].

Then I is a x- C.S.P.I .

Remark

If N is a near integral domain .{0} may be not 0- C.S.P.I as in the following example .

Example (2.13)

Consider the integral domain of the integers Z .The ideal I ={0} is not C.S.P.I of Z .Since

$0 \cdot 5^2 \in I$ **but** $5 \notin I$

Definition (2.14)

The near ring N is called x- completely semi prime ideal near ring denoted by (x- C.S.P.I near ring), if every ideal of a near ring N is x- C.S.P.I of N .

Example (2.15)

consider the near ring in example (2.2)

The ideals of N are I_1={0,a} ,I_2=N ,I_3={0} are b - C.S.P.I of N since $\forall y \in N$ $\mathrm{b} \cdot \mathrm{y}^2 \in \mathrm{I_i}$ implies $\mathrm{y} \in \mathrm{I_i}$, $\mathrm{y,b} \in \mathrm{N}$ and $\mathrm{i} \in \{1,2,3\}$

then N is b- C.S.P.I near ring .

Theorem (2.16)

Let $\{N_j\}_{j \in J}$ **be a family of a near rings,** $x_j \in N_j$ **and I_j be x_j - C.S.P.I for all** $j \in J$ **.Then** $\prod_{i \in J} I_j$ **is (x_j)- C.S.P.I of the direct product near ring** $\prod_{i \in J} N_j$ **.**

Proof

$Let\ (y_j) \in \prod_{j \in J} N_j$ such that

$(\mathrm{x_j})(y_j)^2 \in \prod_{j \in J} I_j$

$\Rightarrow (\mathrm{x_j})(y_j)^2 = (\mathrm{x_j} y_j{}^2) \in \prod_{j \in J} I_j$

$\Rightarrow \mathrm{x_j} y_j{}^2 \in I_j,\ for$ all $\mathrm{j} \in \mathrm{J}$

$\Rightarrow y_j \in I_j$[since each I_j is $\mathrm{x_j} - C.S.P.I$]

$\Rightarrow (y_j) \in \prod_{j \in J} I_j$

$\Rightarrow \prod_{j \in J} I_j$ is $(\mathrm{x_j}) - C.S.P.I$

Corollary (2.17)

Let $\{N_j\} j \in J$ **be a family of** x_j **- C.S.P.I near rings where** $x_j \in N_j$ **for all** $j \in J$ **.Then the product near ring** $\prod_{i \in J} N_j$ **is (x_j)- C.S.P.I.**

Proof

Let I be an ideal of the product near ring $\prod_{i \in J} N_j \Rightarrow$ there exist a family of ideals

$\{I_j\}_{j \in J}$ such that $I = \prod_{j \in J} I_j$ and each I_j is an ideal of a near ring N_j ,for all $j \in J \Rightarrow$ each I_j

x_j - C.S.P.I of N_j , ,for all $j \in J$.[since N_j is x_j - C.S.P.I ,for all $j \in J$].

Now by proposition (2.16)

We have $\prod_{j \in J} I_j = I$ is $(\mathrm{x_j}) - C.S.P.I$

Of the product near ring $\Rightarrow \prod_{j \in J} N_j$ is a $(\mathrm{x_j}) - C.S.P.I$ near ring

Corollary (2.18)

Let I be an ideal of the x- C.S.P.I near ring N.Then the factor near ring N/I is x+I - C.S.P.I ring .

proof

The natural homomorphism $\mathrm{nat}_I : N \to N/I$ which is defined by nat$_I$ (a) =a+I , for all $a \in N$

Is an epimomorphism .Now let J be an ideal of the factor near ring N/I .Then by theorem (2.11) we have $nat_I^{-1}(J)$ is an ideal of the near ring N. $\Rightarrow nat_I^{-1}(J)$ is a x-C.S.P.I of N [since N is x-C.S.P.I near ring .By theorem (2-12) we have $nat(nat_I^{-1}(J)) = J$ is nat$_I$ (x) -C.S.P.I of N/I $\Rightarrow$ J is x+I –C.S.P.I of factor near ring .Then N/I is x+I - C.S.P.I ring.

Remark

When N is a ring all our results are true since our definitions dependent on the multiplication .

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الخلاصة

قدمنا في هذا البحث مفهومين جديدين هما المثالية الشبه الأولية تامة بالنسبة لعنصر في الحلقة القريبة والذي يرمز لها (x-C.S.P.I) وايضا الحلقة القريبة الشبه تامة بالنسبة للعنصر x كما درسنا الصور المباشرة ومعكوس الصور للمثالية (x-C.S.P.I) تحت التشاكل الشامل وحاصل الضرب الديكارتي للحلقات القريبة الشبه تامة بالنسبة للعنصر x .