Bifurcation Solutions of a System of Nonlinear Differential Equations

Ibrahim A. Shanan, Mudhir A. Abdul Hussain

Department of Mathematics, College of Education for Pure sciences,

University of Basrah

Abstract

This paper study the bifurcation solutions of a System of nonlinear differential equations, by using local method of Lyapunov –Schmidt . The reduced equation has been found as a system of nonlinear algebraic equations . We gave a Geometric description of The Discriminate set with the spreading of the regular solutions of a specified system.

1. Introduction.

It is known that many of the nonlinear problems that appear in Mathematics and Physics can be written in the form of operator equation,

$$F(x,\lambda) = b, \quad x \in \overline{U} \subset X, \quad b \in Y, \quad \lambda \in R^n \qquad (1.1)$$

where $\boldsymbol{F}$ is a smooth Fredholm map of index zero, X, Y are real Banach spaces and $\overline{U}$ is open subset of X. For these problems, the method of the reduction to finite dimensional equation,

$$\Theta(\xi,\lambda) = \beta, \quad \xi \in \widehat{M}, \quad \beta \in \widehat{N}. \qquad (1.2)$$

can be used, where $\hat{M}$ and $\hat{N}$ are smooth finite dimensional manifolds.Vainberg and Trenogin (1969) [8], Loginov (1985) [3] and Sapronov (1973, 1996) [11,12] are dealing with equation (1.1) into equation (1.2) by using local method of Lyapunov –Schmidt with the conditions that, equation (1.2) has all the topological and analytical properties of equation (1.1) (multiplicity, bifurcation diagram, *etc*).

Definition 1.1 Suppose that E and M are Banach spaces and $A : E \to M$ be a linear continuous operator. The operator A is called Fredholm operator, if

1- The kernel of A, *Ker*(A), is finite dimensional,

2- The range of A, Im(A), is closed in M,

3- The Cokernel of A, Coker(A), is finite dimensional.

The number

dim(*Ker A*)- *dim*(*Coker A*)

is called Fredholm index of the operator A.

Definition 1.2 The set of all λ for which equation (1.1) has degenerate solutions is called the Discriminate set of equation (1.1) and denoted it by $\sum$.

The oscillations and motion of wave on elastic foundations can be described by the following nonlinear PDE

$$\frac{d^4w}{dx^4} + \alpha\frac{d^2w}{dx^2} + \beta w + \mathrm{g}(\lambda,\widetilde{w}) = 0, \quad (1.3)$$

$$\widetilde{w} = (w, w^{/}, w^{//}, w^{///}, w^{////}).$$

where $\alpha, \beta > 0$. Equation (1.3) has many studies When $g(\lambda, \tilde{w}) = -k\,w^3$, Bardin and Furta [1] used the local method of Lyapunov-Schmidt and found the sufficient conditions of existence of periodic waves of equation (1.3). Equation (1.3) also has been studied by Thompson and Stewart [4]. They showed numerically the existence of periodic solutions of equation (1.3) for some values of parameters. Mohammed [7] studied equation (1.3) in the variational case when $g(\lambda, \tilde{w}) = w^2 + w^3$. Equation (1.3) has been studied by Sapronov [2,10]. He applied the local method of Lyapunov – Schmidt and found the bifurcation solutions of equation (1.3) when $g(\lambda, \tilde{w}) = w^3$ with the boundary conditions,

$$w(0) = w(\pi) = w^{//}(0) = w^{//}(\pi) = 0.$$

Abdul Hussain (2005, 2009) [5, 6], he found the bifurcation solutions of equation (1.3) by using local method of Lyapunov-Schmidt. Shanan [13] solved the bifurcation equation corresponding to the equation (1.3) and found the bifurcation diagram of this problem, when $g(\lambda, \tilde{w}) = w^2$, $g(\lambda, \tilde{w}) = w^2 + w^3$ and when the dimension of the null space is equal to three.

In this work we considered the system of non-linear differential equations of the fourth order,

$$\frac{d^4u_1}{dx^4} + \lambda_1 \frac{d^2u_1}{dx^2} + u_1 + u_1^3 + u_1u_2^2 + u_1^2u_3 + u_3^3 = \psi_1$$

$$\frac{d^4u_2}{dx^4} + \lambda_2 \frac{d^2u_2}{dx^2} + u_2 + u_1^3 + u_1u_3^2 + u_1^2u_2 + u_2^3 = \psi_2 \qquad (1.4)$$

$$\frac{d^4u_3}{dx^4} + \lambda_3 \frac{d^2u_3}{dx^2} + u_3 + u_1u_2^2 + u_1u_3^2 + u_2^3 + u_3^3 = \psi_3$$

with the boundary conditions

$$u_1(0) = u_1(\pi) = u_1^{//}(0) = u_1^{//}(\pi) = 0.$$

$$u_2(0) = u_2(\pi) = u_2^{//}(0) = u_2^{//}(\pi) = 0.$$

$$u_3(0) = u_3(\pi) = u_3^{//}(0) = u_3^{//}(\pi) = 0.$$

where $\lambda_1, \lambda_2, \lambda_3 \in R$ and ψ_i, $i = 1,2,3$, are symmetric functions with respect to the involutions I_i: $\psi_i(x) \to \psi_i(\pi - x)$.

2. Reduction to Bifurcation Equation.

Consider the following nonlinear system of ODEs,

$$AU + \text{Ɲ}U = \psi \qquad (2.1)$$

with the boundary conditions

$$U(0) = U(\pi) = U^{//}(0) = U^{//}(\pi) = 0.$$

$$where \quad A = \frac{d^4}{dx^4} + \frac{d^2}{dx^2}B + I, \qquad B = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$\text{Ɲ}U = \begin{pmatrix} u_1^3 + u_1u_2^2 + u_1^2u_3 + u_3^3 \\ u_1^3 + u_1u_3^2 + u_1^2u_2 + u_2^3 \\ u_1u_2^2 + u_1u_3^2 + u_2^3 + u_3^3 \end{pmatrix}, \quad U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

ψ is a symmetric vector function with respect to the involution $I : \psi(x) \mapsto \psi(\pi - x)$.Our goal is to study the bifurcation solutions of system (2.1) by using local method of Lyapunov –Schmidt. To do this , we first put equation (2.1) in the form of operator equation, *i.e*

$$F(U, \lambda) = AU + \text{Ɲ}U \quad (2.2)$$

where $F : E \to M$ is a nonlinear Fredholm operator of index zero from Banach space E to Banach space M, $E = C^4([0,\pi], R^3)$, $M = C^0([0,\pi], R^3)$, $x \in [0,\pi]$. The

solutions of equation (2.1) are in one-to-one corresponding with the solutions of the following equation

$F(U,\lambda)=\psi$ (2.3)

Now ,we prove the reduction theorem of equation (2.1).

Theorem 2.1 The bifurcation equation corresponding to problem (2.1) is given by the following nonlinear system of three cubic equations,

$$\begin{pmatrix} \frac{3}{2\pi}x_1^3+\frac{1}{\pi}x_1x_2^2-\frac{1}{2\pi}x_1^2x_3+k_1x_1-q_1 \\ \frac{1}{\pi}x_1^2x_2+\frac{3}{2\pi}x_2^3+k_2x_2 \\ \frac{3}{2\pi}x_3^3+\frac{1}{2\pi}x_1x_2^2+k_3x_3-q_3 \end{pmatrix} + o(|\tau|^3)+O(|\tau|^3)O(\gamma)=0.$$

where, $\tau=(x_1,x_2,x_3)$,

$q_1,q_3,k_1,k_2,k_3\in R,\quad \gamma=(\delta_1,\delta_2,\delta_3)$

Furthermore, the solutions of the equation in (2.1) are in one-to-one corresponding with the solutions of the nonlinear system.

Proof: Let

$F(U,\lambda)=\psi$ where $\psi\in M$ (2.4)

Since, the Frechet derivative of the nonlinear operator $F(U,\lambda)$ at the point $(0,\lambda)$ is

$$dF(0,\lambda)h=\frac{d^4}{dx^4}h+\frac{d^2}{dx^2}Bh+Ih,$$

Where $\lambda=(\lambda_1,\lambda_2,\lambda_3)$, $h=(h_1,h_2,h_3)^{T}$

then the linearized equation corresponding to the equation (2.4) is given by,

$$Ah=0\ ,\qquad h\in E,$$
$$A=dF(0,\lambda)=\frac{d^4}{dx^4}+\frac{d^2}{dx^2}B+I,$$

The solution of linearized equation which satisfied the boundary conditions is given by,

$$h(x)=\begin{pmatrix} c_1sin(px) \\ c_2sin(qx) \\ c_2sin(lx)\end{pmatrix}$$

Substitute in the linearized equation we have characteristic equation corresponding to this solution in the form ,

$$p^4-\lambda_1p^2+p=0$$
$$q^4-\lambda_2q^2+q=0$$
$$l^4-\lambda_3l^2+l=0$$

This equation gives in the space of parameters $(\lambda_1,\lambda_2,\lambda_3)$ characteristic planes ℓ_p. The characteristic planes ℓ_p consists the points $(\lambda_1,\lambda_2,\lambda_3)$ for which the linearized equation has non-zero solutions. The point of intersection of characteristic planes in the space of parameters $(\lambda_1,\lambda_2,\lambda_3)$ is a bifurcation point .

So for equation (2.4) the point $(\lambda_1,\lambda_2,\lambda_3)=(2\,,18/4\ ,\ 84/9)$ is a bifurcation point.

Localized parameters $\lambda_1,\lambda_2,\lambda_3$ as follows,

$\lambda_1=2+\delta_1,\quad \lambda_2=18/4+\delta_2,\quad \lambda_3=84/9+\delta_3,\quad \delta_1,\delta_2,\delta_3$ are small parameters.

lead to bifurcation along the modes

$e_1(x)=(c_1\sin(x)\,,0\,,0)^{T}$, $e_2(x)=(0\,,c_2\sin(2x),0)^{T}$,

$e_3(x)=(0\,,0\,,c_3\sin(3x))^{T}$

Where $\|e_1\|=\|e_2\|=\|e_3\|=1$

It is easy to see that for $n,m=1,2,3$ and

$$c_1 = c_1 = c_1$$

$$= \sqrt{\frac{2}{\pi}} \ , \qquad \int_0^{\pi} \sin(nx)\sin(mx)\,dx$$

$$= \begin{cases} 1 \ if \ \ n = m \\ 0 \ if \ n \neq m \end{cases}$$

this mean that e_1, e_2 and e_3 are orthonormal basis of the null space $Ker(A)$.

Let $N = Ker(A) = span\{e_1, e_1, e_1\}$, then the space E can be decomposed in direct sum of two subspaces, N and the orthogonal complement to N,

$$E = N \oplus N^{\perp} \quad , \quad N^{\perp} = \{z \in E: \int_0^{\pi} z e_k dx = 0,\ k = 1,2,3\}.$$

Similarly, the space M can be decomposed in direct sum of two subspaces, N and orthogonal complement to N,

$$M = N \oplus \widetilde{N}^{\perp} \quad , \quad \widetilde{N}^{\perp} = \{v \in E: \int_0^{\pi} v e_k dx = 0 \ , k = 1,2,3\}.$$

There exist two projections $P: E \to N$ and $(I - P): E \to N^{\perp}$ such that

$$PU = W \ and \ (I - P)U = Z.$$

Where $U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $W = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, $Z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$

Hence every vector $U \in E$ can be written in the unique form,

$$U = W + Z$$

$$W = \sum_{i=1}^{3} x_i e_i \ \in \ker(A) \ , \ Z \perp \ker(A) , \quad x_i = \langle U, e_i \rangle$$

Similarly, there exist two projections $Q: M \to N^{\perp}$ and $(I - Q): M \to \widetilde{N}^{\perp}$ such that

$$QF(U,\lambda) = F_1(U,\lambda),$$

$$(I - Q)F(U,\lambda) = F_2(U,\lambda),$$

and hence

$$F(U,\lambda) = F_1(U,\lambda) + F_2(U,\lambda)$$

$$= QF(U,\lambda) + (I - Q)F(U,\lambda),$$

$$F_1(U,\lambda) = QF(U,\lambda) = \sum_{i=1}^{3} v_i\,(U,\lambda) e_i \in N,$$

$$F_2(U,\lambda) = (I - Q)F(U,\lambda) \in \ \widetilde{N}^{\perp},$$

$$v_i(U,\lambda) = \langle F(U,\lambda), e_i \rangle$$

Accordingly, equation (2.2) can be written in the form,

$$QF(U,\lambda) = \tilde{\psi}_1,$$

$$(I - Q)F(U,\lambda) = \tilde{\psi}_2$$

or

$$QF(W + Z,\lambda) = \tilde{\psi}_1$$

$$(I - Q)F(W + Z,\lambda) = \tilde{\psi}_2$$

Where $\psi = \tilde{\psi}_1 + \tilde{\psi}_2$, $\tilde{\psi}_1 = q_1 e_1 + q_2 e_2 + q_3 e_3 \in N$, $\tilde{\psi}_2 \in \ \widetilde{N}^{\perp}$.

By the implicit function theorem, there exists a smooth map $\Theta : N \to N^{\perp}$ such that $\Theta(W,\lambda) = Z$ and

$$(I - Q)F(W + \Theta(W,\lambda),\lambda) = \tilde{\psi}_2,$$

To find the solutions of the equation (2.2) in the neighbourhood of the point $U = 0$ it is sufficient to find the solutions of the equation,

$$QF(W + \Theta(W,\lambda),\lambda) = \tilde{\psi}_1 \quad (2.5)$$

and then we have the bifurcation equation in the form,

$$\Phi(\tau,\lambda) = \tilde{\psi}_1, \ \ \tau = (x_1, x_2, x_3), \ \ \lambda = (\lambda_1, \lambda_2, \lambda_3),$$

where

$$\Phi(\tau,\lambda) = F_1(W + \Theta(W,\lambda),\lambda).$$

Equation (2.4) can be written in the fo

$$F(W + Z,\lambda) = A(W + Z) + \mathrm{N}(W + Z) = AW + \mathrm{N}W + \ \ldots$$

Where the dots denote the terms that consists of the element Z. Hence,

$$\Phi(\tau,\lambda) = QF(W+Z,\lambda) = \sum_{i=1}^{3}\langle AW+\text{N}W, e_i\rangle + \cdots = \tilde{\psi}_1 \qquad (2.6)$$

where $\langle .,.\rangle$ is the scalar product in Hilbert space $L_2([0,\pi].R)$.

Equation (2.6) implies that,

$\langle AW+\text{N}W,e_1\rangle e_1 + \langle AW+\text{N}W,e_2\rangle e_2 + \langle AW+\text{N}W,e_3\rangle e_3 + \cdots = \psi_1$ (2.7)

Simple calculations of equation (2.5) implies that,

$$\langle AW+\text{N}W,e_1\rangle = \frac{3}{2\pi}x_1^3 + \frac{1}{\pi}x_1x_2^2 - \frac{1}{2\pi}x_1^2x_3 + k_1x_1 ,$$

$$\langle AW+\text{N}W,e_2\rangle = \frac{1}{\pi}x_1^2x_2 + \frac{3}{2\pi}x_2^3 + k_2x_2$$

and

$$\langle AW+\text{N}W,e_3\rangle = \frac{3}{2\pi}x_3^3 + \frac{1}{2\pi}x_1x_2^2 + k_3x_3$$

where

$$k_1 = Ae_1 = \alpha_1(\lambda)e_1, \qquad k_2 = Ae_2 = \alpha_2(\lambda)e_2, \qquad k_3 = Ae_3 = \alpha_1(\lambda)e_3.$$

Symmetry of the function $\psi(x)$ with respect to the involution

$I:\psi(x) \to \psi(\pi - x)$ implies that $q_2 = 0$ and hence we have

$$\left(\frac{3}{2\pi}x_1^3 + \frac{1}{\pi}x_1x_2^2 - \frac{1}{2\pi}x_1^2x_3 + k_1x_1\right)e_1 + \left(\frac{1}{\pi}x_1^2x_2 + \frac{3}{2\pi}x_2^3 + k_2x_2\right)e_2 + \left(\frac{3}{2\pi}x_3^3 + \frac{1}{2\pi}x_1x_2^2 + k_3x_3\right)e_3 + \cdots = q_1e_1 + q_3e_3$$

By equating the coefficients of e_1, e_2 and e_3 we have,

$$\begin{pmatrix} \frac{3}{2\pi}x_1^3 + \frac{1}{\pi}x_1x_2^2 - \frac{1}{2\pi}x_1^2x_3 + k_1x_1 - q_1 \\ \frac{1}{\pi}x_1^2x_2 + \frac{3}{2\pi}x_2^3 + k_2x_2 \\ \frac{3}{2\pi}x_3^3 + \frac{1}{2\pi}x_1x_2^2 + k_3x_3 - q_3 \end{pmatrix} + o(|\tau|^3) + O(|\tau|^3)O(\gamma) = 0. \qquad (2.8)$$

3. Analysis of bifurcation.

The reduced system (2.8) has all the information we need from the Lyapunov-Schmidt, it has all the topological and analytical properties of equation in (2.2). So to study bifurcation solutions of equation in (2.2) it is sufficient to the study bifurcation solutions of system (2.8).

Let

$$\Phi(\tau,\tilde{\lambda}) = \begin{pmatrix} \frac{3}{2\pi}x_1^3 + \frac{1}{\pi}x_1x_2^2 - \frac{1}{2\pi}x_1^2x_3 + k_1x_1 - q_1 \\ \frac{1}{\pi}x_1^2x_2 + \frac{3}{2\pi}x_2^3 + k_2x_2 \\ \frac{3}{2\pi}x_3^3 + \frac{1}{2\pi}x_1x_2^2 + k_3x_3 - q_3 \end{pmatrix} + o(|\tau|^3) + O(|\tau|^3)O(\gamma),$$

then the Discriminant set $\sum$ of the map $\Phi(\tau,\tilde{\lambda})$ is locally contact equivalent in the neighborhood of the point zero to the Discriminant set of the map $\Phi_1(\tau,\tilde{\lambda})$ [9],

$$\Phi_1(\tau,\tilde{\lambda}) = \begin{pmatrix} x_1^3 + x_1x_2^2 - x_1^2x_3 + k_1x_1 - q_1 \\ x_1^2x_2 + x_2^3 + k_2x_2 \\ x_3^3 + x_1x_2^2 + k_3x_3 - q_3 \end{pmatrix} \qquad (3.1)$$

$q_1, q_3, k_1, k_2, k_3 \in R$

this means that, to determine the Discriminant set of the map $\Phi(\tau,\tilde{\lambda})$ it is sufficient to determine the Discriminant set of the map (3.1).

Let

$$\widehat{\Phi}_1(\tau,\tilde{\lambda}_1) = \begin{pmatrix} x_1^3 - x_1^2x_3 + k_1x_1 - q_1 \\ x_3^3 - k_3x_3 - q_3 \end{pmatrix} \qquad (3.2)$$

and

$$\widehat{\Phi}_2(\tau,\tilde{\lambda}_2) = \begin{pmatrix} -x_1^2x_3 - k_2x_1 + k_1x_1 - q_1 \\ x_3^3 - x_1^3 - k_2x_1 + k_3x_3 - q_3 \end{pmatrix} \qquad (3.3)$$

then the solution set of equation $\Phi_1(\tau,\tilde{\lambda})=0$ is decomposed into following two sets, $S_1=\{(x_1,0,x_3):\ x_1,x_3\in D_1\}$,

$$S_2=\{(x_1,x_2,x_3):\ x_2^2=-x_1^2-k_2\,,\ x_1,x_3\in D_2\}$$

where D_1 and D_2 are the solution sets of the equations $\widehat{\Phi}_1(\tau,\tilde{\lambda}_1)$ and $\widehat{\Phi}_2(\tau,\tilde{\lambda}_2)$, respectively. In this case the Discriminant set $\square$ of equation $\Phi_1(\tau,\tilde{\lambda})$ is the union of the Discriminant sets $\square_1$ and $\square_2$ of the equations $\widehat{\Phi}_1(\tau,\tilde{\lambda}_1)=0$ and $\widehat{\Phi}_2(\tau,\tilde{\lambda}_2)=0$, respectively ($\Sigma=\Sigma_1\cup\Sigma_2$). The points set S_1 is degenerate on the surface defined by the equation

$$det\left(\frac{\partial\Phi_1(\tau,\tilde{\lambda})}{\partial x_i}\right)\Bigg|_{x_2=0}=0 \qquad i=1,2,3 \qquad \text{... (3.4)}$$

equation (3.4) has two solutions with respect to the variable x_1 . By using these solutions we can obtain the Discriminant set $\Sigma_1$of equation $\hat{\Phi}_1(\tau,\tilde{\lambda}_1)=0$ in plane of parameters q_1 and q_3. The points set S_2 is degenerate on the surface defined by the equation

$$det\left(\frac{\partial\Phi_1(\tau,\tilde{\lambda})}{\partial x_i}\right)\Bigg|_{x_2=\sqrt{-x_1^2-k_2}}=0 \qquad i=1,2,3 \qquad \text{... (3.5)}$$

If we solved equation (3.5) with respect to the variable $x_1$we have four solutions. By using these solutions and take the union $\Sigma=\Sigma_1\cup\Sigma_2$ we can find the final form of the Discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})=0$. The final form of the Discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})=0$ has been found in the q_1q_3-plane for some values of $k_1,k_2 \, and \, k_3$ with the number of regular solutions in every region in the following figures,

The figures have been drawing by using Maple 13.

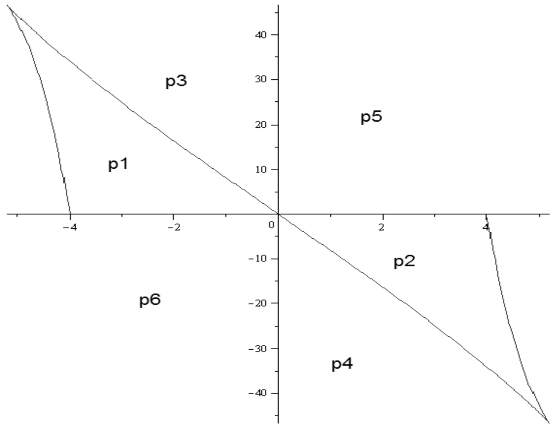

Fig. 1 The Discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})=0$when $k_1=9,k_2=0.007,k_3=-36$

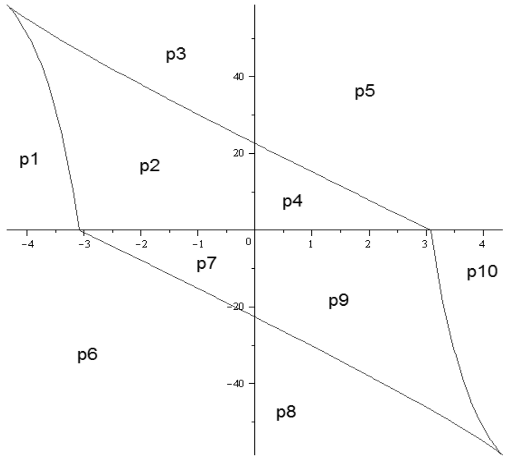

Fig. 2 The Discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})=0$ when $k_1=8,k_2=0.007,k_3=-36$

Figure (1) describes the discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})$ in the q_1q_3-plane when $k_1=9,k_2=0.007,k_3=-36$.

The Discriminant set decompose the solution set into regions q_i, $i=1,\dots,6$, every region has only five regular solutions with topological indices 1,-1,1,-1,1.

Figure (2) describes the discriminant set of equation $\Phi_1(\tau,\tilde{\lambda})$ in the q_1q_3-plane when $k_1=8,k_2=0.007,k_3=-36$,The equation has no more than seven solutions in every region. The spreading of regular solutions is given as follows:

(i) In the regions p_2, p_4, p_7 *and* p_9 the equation has seven regular solutions for every $\tilde{\lambda} \in p_i$, $i = 2,4,7,9$ with topological indices 1,-1,1,-1,1,-1,1

(ii) In the regions p_1, p_3, p_8 *and* p_{10} the equation has five regular solutions for ever $\tilde{\lambda} \in p_i$, $i = 1,3,8,10$ with topological indices 1,-1,1,-1,1

(iii) In the regions p_5 *and* p_6 the equation has three regular solutions for ever $\tilde{\lambda} \in p_i$, $i = 5,6$ with topological indices 1,-1,1

References:

[1] B.C. Bardin, S.D. Furta, Periodic travelling waves of an infinite beam on a nonlinear elastic support, Institute B für Mechanik, Universität Stuttgart, Institutsbericht IB-36. Januar 2001.

[2] B.M. Darinskii, C.L. Tcarev, Yu.I. Sapronov, Bifurcation of extremely of Fredholm functionals, Voronezh, 2004.

[3] B.V. Loginov, Theory of Branching nonlinear equations in theconditions of invariance group, - Tashkent: Fan, 1985.

[4] J. M. T. Thompson, H. B. Stewart, Nonlinear Dynamics and Chaos, Chichester, Singapore, J. Wiley and Sous, 1986.

[5] M.A. Abdul Hussain, Corner singularities of smooth functions in the analysis of bifurcations balance of the elastic beams and periodic waves, Ph. D. thesis, Voronezh-Russia. 2005.

[6] M.A. Abdul Hussain, Bifurcation solutions of elastic beams equation with small perturbation, Int. journal of Mathematical Analysis, Vol. 3, no. 18, 2009, 879-888.

[7] M.J. Mohammed, Bifurcation solutions of nonlinear wave equation, M.Sc. thesis, Basrah Univ., Iraq, 2007.

[8] M.M. Vainberg, V.A. Trenogin, Theory of Branching solutions of nonlinear equations, M.-Science , 1969.

[9] V.I. Arnold, Singularities of Differential Maps, M., Sience, 1989.

[10] V.R. Zachepa, Yu.I. Sapronov, Local analysis of Fredholm equation, - Voronezh, 2002.

[11] Yu.I. Sapronov, Regular perturbation of Fredholm maps and theorem about odd field, Works Dept. of Math., Voronezh Univ., 1973. V. 10, 82-88.

[12] Yu.I. Sapronov, Finite dimensional reduction in the smooth extremely problems, - Uspehi math., Science, - 1996, V. 51, No. 1., 101- 132.

[13] A.K. Shanan , Three Modes Bifurcation Solutions of Nonlinear Fourth order Differential Equation , Basrah, IRAQ,2011.

المستخلص

درسنا في هذا البحث حلول التفرع لنظام من المعادلات التفاضلية الغير خطية باستخدام طريقة ليابونوف – شمدت المحلية . عند تخفيض النظام حصلنا على نظام من المعادلات الجبرية الغير خطية ، كذلك تم ايجاد وصف هندسي للمجموعة المميزة مع انتشار الحلول المنتظمة للنظام.

ابراهيم عبد علي شنان ، مظهر عبد الواحد عبد الحسين

جامعة ألبصرة- كلية التربية للعلوم ألصرفة- قسم الرياضيات