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On Artin's characters table of the group ($Q_{2m} \times C_p$) when m is an odd number and p is prime number

حول جدول شواخص ارتن للزمرة ($Q_{2m} \times C_p$) عندما m عدد فردي و p عدد اولي

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Abstract---

In this paper, we prove that the general form of Artin's characters table of the group ($Q_{2m} \times C_p$) such that Q_{2m} be the Quaternion group of order 4m when m is an odd number and C_p be the cyclic group of order p when p is prime number and ($Q_{2m} \times C_p$) be direct product of Q_{2m} and C_p such that

$$(Q_{2m} \times C_p) = \{(q,c): q \in Q_{2m}, c \in C_p\} \text{ and } |Q_{2m} \times C_p| = |Q_{2m}|.|C_p| = 4m.p = 4pm.$$

This table which depends on Artin's characters table of a quaternion group of order 4m when m is an odd number. which is denoted by Ar($Q_{2m} \times C_p$).

المستخلص---

في هذا البحث سوف نقوم ببرهان الشكل العام لجدول شواخص ارتن للزمرة ($Q_{2m} \times C_p$) بحيث Q_{2m} هي الزمرة الرباعية العمومية ذات الرتبة 4m عندما m عدد فردي و Cp هي الزمرة الدائرية ذات الرتبة p عندما p عدد اولي و($Q_{2m} \times C_p$) هي الضرب المباشر للزمرة Q_{2m} و الزمرة C_p بحيث ان

$$|Q_{2m} \times C_p| = |Q_{2m}|.|C_p| = 4m.p = 4pm. \text{ و } (Q_{2m} \times C_p) = \{(q,c): q \in Q_{2m}, c \in C_p\}$$

هذا الجدول يعتمد على جدول شواخص ارتن للزمرة الرباعية العمومية ذات الرتبة 4m عندما m عدد فردي . ويرمز لهذا الجدول بالشكل .Ar($Q_{2m} \times C_p$)

Key words--the Quaternion group, the group ($Q_{2m} \times C_p$), Artin's characters table, the cyclic subgroup.

1. INTRODUCTION

Let G be a finite group, two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes.let R(G) denotes the abelian group generated by Z-valued characters of G under the operation of point wise addition. Inside this group there is a subgroup generated by Artin characters (The characters induced form the principal characters of cyclic subgroups of G). In 1967, T.Y. Lam [9] prove a sharp form of Artin theorem and he determine the least positive integer $A(G)$ such that [$\overline{R}$ (G):(G)]=A(G). In1976, I. M. Isaacs [4] studied Character Theory of Finite Groups. In 2008, A.H. Abdul-Mun'em [1] studied the Artin cokernel of the Quaternion Group Q_{2m} when m is an odd Number.

The aim of this paper is to find the general from of the Artin's characters table of the group ($Q_{2m} \times C_p$) when m is odd number and p is a prime number.

2. PRELIMINARIES

This section introduce some important definitions and basic concepts of the induced character, the Artin's characters tables, the Artin's characters table of C_{p^s} and the Artin's characters table of the Quaternion group Q_{2m} when m is an odd number.

Definition (2.1): [8]

Two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G , this defines an equivalence relation on G. It is classes are called Γ- classes.

Example (2.2):

Consider a cyclic group C_{11}=< x > such that:
1 is Γ- conjugate 1
Then the Γ- class [1] = {1}
$< x > = < x^2 > = < x^3 > = < x^4 > = < x^5 > = < x^6 > = < x^7 > = < x^8 > = < x^9 > = < x^{10} >$
Then $x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9$ and x^{10} are Γ-conjugate , and $[x] = \{x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}\}$
So that there are two Γ- classes of C_{11} :[1] and [x]
In general for C_{p^s} where p is any prime number, so that are s+1 distinct Γ-classes which are [1], [x], [x^p], … ,[$x^{p^{s-1}}$] .

Definition (2.3):[4]

Let H be a subgroup of G and let φ be a class function on H, ***the induced class function on G***, is given by:

$$\varphi'(g) = \frac{1}{|H|}\sum_{h\in G}\varphi^{\circ}(hgh^{-1})$$

where φ° is defined by:

$$\varphi^{\circ}(x) = \begin{cases}\varphi(x) & if \quad x \in H \\ 0 & if \quad x \notin H\end{cases}$$

Proposition (2.4) :[7]

Let *H* be a subgroup of *G* and φ be a character of *H*, then φ' is a character of *G*.

Definition (2.5):[5]

The character φ' of *G* is called ***induced character*** on *G*.

Theorem (2.6):[3]

Let *H* be a cyclic subgroup of *G* and $h_1, h_2, \ldots, h_m$ are chosen representative for *m*-conjugate classes, then :

1- $\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$; if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$; if $H \cap CL(g) = \phi$.

Definition (2.7):[4]

Let *G* be a finite group, all characters of *G* induced from a principal character of cyclic subgroups of *G* are called ***Artin's characters of G.***

Proposition (2.8):[3]

The number of all distinct Artin's characters on a group *G* is equal to the number of Γ-classes on G. furthermore, Artin's characters are constant on each Γ-classes.

Definition(2.9):[2]

Artin's characters of finite group *G* can be displayed in table ***called Artin's characters table of G*** which is denoted by Ar (*G*).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest row contain the values of Artin's characters.

Example (2.10):

In the Artin's character table of C_{11} there are two Γ- classes, [1] and [x] then, two distinct Artin's characters can be obtained by proposition (2.8), we get the following table.

Table (1)

Ar (C_{11}) =

Γ- classes	[1]	[x]
$\lvert CL_\alpha\rvert$	1	1
$\lvert C_{C_{11}}(CL_\alpha)\rvert$	11	11
φ'_1	11	0
φ'_2	1	1

Theorem(2.11):[2]

The general form of Artin's character table of C_{p^s} when *p* is a prime number and *s* is an integer number is given by:

Ar(C_{p^s})=

Table(2)

Γ- classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^p]$	$[x]$
$\lvert CL_\alpha\rvert$	1	1	1	1	…	1	1
$\lvert C_{p^s}(CL_\alpha)\rvert$	P^s	p^s	P^s	p^s	…	p^s	P^s
φ'_1	p^s	0	0	0	…	0	0
φ'_2	P^{s-1}	P^{s-1}	0	0	…	0	0
φ'_3	P^{s-2}	P^{s-2}	P^{s-2}	0	…	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮
φ'_s	P	P	P	P	…	P	0
φ'_{s+1}	1	1	1	1	…	1	1

Example (2.12):

Consider the cyclic group C_{16}, To find the Artin's character table we use theorem (2.11) as follows: the group $C_{16} = C_{2^4}$ then:

Ar (C_{2^4}) =

Table (3)

Γ- classes	[1]	$[x^{2^3}]$	$[x^{2^2}]$	$[x^2]$	[x]
$\lvert CL_\alpha\rvert$	1	1		1	1
$\lvert C_{2^4}(CL_\alpha)\rvert$	2^4	2^4	2^4	2^4	2^4
φ'_1	2^4	0	0	0	0
φ'_2	2^3	2^3	0	0	0
φ'_3	2^2	2^2	2^2	0	0
φ'_4	2	2	2	2	0
φ'_5	1	1	1	1	1

Theorem (2.13): [1]

The Artin's characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

Table(4)

$Ar(Q_{2m}) =$

Γ- classes	Γ- classes of C_{2m}								
	x^{2r}				x^{2r+1}				[y]
$\lvert CL_\alpha \rvert$	1	2	…	2	1	2	…	2	2m
$\lvert C_{Q_{2m}}(CL_\alpha) \rvert$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	**$2Ar(C_{2m})$**								0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	…	0	M	0	…	0	1

Where $0 \leq r \geq m -1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \leq j \leq l+1$.

Example (2.14):

To construct $Ar(Q_{30})$ by using theorem (1.13), we get the following table :

Table (5)

$Ar(Q_{30})=$

Γ- classes	[1]	[x^{10}]	[x^6]	[x^2]	[x^{15}]	[x^5]	[x^3]	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	2	2	1	2	2	2	30
$\lvert C_{Q_{30}}(CL_\alpha) \rvert$	60	30	30	30	60	30	30	30	2
Φ_1	60	0	0	0	0	0	0	0	0
Φ_2	20	20	0	0	0	0	0	0	0
Φ_3	12	0	12	0	0	0	0	0	0
Φ_4	4	4	4	4	0	0	0	0	0
Φ_5	30	0	0	0	30	0	0	0	0
Φ_6	10	10	0	0	10	10	0	0	0
Φ_7	6	0	6	0	6	0	6	0	0
Φ_8	2	2	2	2	2	2	2	2	0
Φ_9	15	0	0	0	15	0	0	0	1

3. THE MAIN RESULTS

In this section we find the general form of Artin's characters table of the group $(Q_{2m}\times C_p)$ when m is an odd number and p is prime number. As in the following Proposition:

Proposition (3.1)

The general form of the Artin's characters table of the group $(Q_{2m}\times C_p)$ when m is an odd number and p is prime number is given as follows:

Table (6)

$Ar(Q_{2m}\times C_p)$ =

<table>
<tr><td></td><td colspan="9">Γ- classes of $(Q_{2m})\times\{I\}$</td><td colspan="9">Γ- classes of $(Q_{2m})\times\{z\}$</td></tr>
<tr><td>***Γ- classes***</td><td colspan="4">$[x^{2r},I]$</td><td colspan="4">$[x^{2r+1},I]$</td><td>$[y,I]$</td><td colspan="4">$[x^{2r},z]$</td><td colspan="4">$[x^{2r+1},z]$</td><td>$[y,z]$</td></tr>
<tr><td>$\lvert CL_\alpha\rvert$</td><td>**1**</td><td>**2**</td><td>…</td><td>**2**</td><td>**1**</td><td>**2**</td><td>…</td><td>**2**</td><td>**2m**</td><td>**1**</td><td>**2**</td><td>…</td><td>**2**</td><td>**1**</td><td>**2**</td><td>…</td><td>**2**</td><td>**2m**</td></tr>
<tr><td>$\lvert C_{Q_{2m}\times C_p}(CL_\alpha)\rvert$</td><td>**4pm**</td><td>**2pm**</td><td>…</td><td>**2pm**</td><td>**4pm**</td><td>**2pm**</td><td>…</td><td>**2pm**</td><td>**2p**</td><td>**4pm**</td><td>**2pm**</td><td>…</td><td>**2pm**</td><td>**4pm**</td><td>**2pm**</td><td>…</td><td>**2pm**</td><td>**2p**</td></tr>
<tr><td>***$\Phi_{(1,1)}$***</td><td colspan="9" rowspan="5">**$pAr(Q_{2m})$**</td><td colspan="9" rowspan="5">**0**</td></tr>
<tr><td>***$\Phi_{(2,1)}$***</td></tr>
<tr><td>⋮</td></tr>
<tr><td>***$\Phi_{(l,1)}$***</td></tr>
<tr><td>***$\Phi_{(l+1,1)}$***</td></tr>
<tr><td>***$\Phi_{(1,2)}$***</td><td colspan="9" rowspan="4">**$Ar(Q_{2m})$**</td><td colspan="9" rowspan="4">**$Ar(Q_{2m})$**</td></tr>
<tr><td>***$\Phi_{(2,2)}$***</td></tr>
<tr><td>⋮</td></tr>
<tr><td>***$\Phi_{(l,2)}$***</td></tr>
</table>

Proof :

Let $g \in (Q_{2m}\times C_p)$;g=(q,I) or g=(q,z)or g=(q,z^2),…g=(q,z^{p-1}), q $\in Q_{2m}$,I,z,z^2,….,z^{p-1} $\in C_p$

<u>*Case (I):*</u>

If H is a cyclic subgroup of $(Q_{2m}\times \{I\})$,then:

1- H=$\langle(x,I)\rangle$
2- 2- H=$\langle(y,I)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (2.6)

1. H= $\langle(x,I)\rangle$

(i) If g=(1,I) and g϶H

$$\Phi_{(j,1)}((1,I))=\frac{\lvert C_{Q_{2m}\times C_p}(g)\rvert}{\lvert C_H(g)\rvert}\cdot\varphi(g)$$

$$=\frac{4pm}{\lvert C_H(I,1)\rvert}.1=\frac{p.4m}{\lvert C_H(I,1)\rvert}.1=\frac{p\lvert C_{Q_{2m}}(1)\rvert}{\lvert C_{\langle x\rangle}(1)\rvert}.\varphi(1)=p.\Phi_j(1)$$

since H ∩CL(1,I)={(1,I)}

(ii) if g= (x^m, I) and g∈H

$$\Phi_{(j,1)}(g)=\frac{\lvert C_{Q_{2m}\times C_p}(g)\rvert}{\lvert C_H(g)\rvert}\cdot\varphi(g)$$

$$=\frac{4pm}{\lvert C_H(g)\rvert}.1=\frac{p.4m}{\lvert C_H(g)\rvert}.1=\frac{p\lvert C_{Q_{2m}}(x^m)\rvert}{\lvert C_{\langle x\rangle}(x^m)\rvert}.\varphi(g)=p.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i, I), i\neq m$ and $i\neq 2m$ and g∈H

$$\Phi_{(j,1)}(g)=\frac{\lvert C_{Q_{2m}\times C_p}(g)\rvert}{\lvert C_H(g)\rvert}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{2pm}{\lvert C_H(g)\rvert}(1+1)=$$

$$\frac{p.2m}{\lvert C_H(g)\rvert}.(1+1)=\frac{p\lvert C_{Q_{2m}}(q)\rvert}{\lvert C_{\langle x\rangle}(q)\rvert}.(\varphi(g)+\varphi(g^{-1}))=p.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)= φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g ∉ H

$$\Phi_{(j,1)}(g)=p.0=p.\Phi_j(q)$$ Since H∩CL(g)= ϕ

2- H=$\langle(y,I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) ; H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{4pm}{4}.1=pm=p.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{4pm}{4}.1=pm=p.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{2p}{4}.(1+1)=p.1=p.\Phi_{l+1}(y)$$

Since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,1)}(g)=0$ since H∩CL(g)=∅

Case (II):

If H is a cyclic subgroup of (Q_{2m}× {z}),then:

1.H=$\langle(x,z)\rangle=\langle(x,z^2)\rangle=\langle(x,z^3)\rangle\ \ldots\ \langle(x,z^{p-1})\rangle$

2.H=$\langle(y,z)\rangle=\langle(y,z^2)\rangle\ \langle(y,z^3)\rangle\ \ldots\ \langle(y,z^{p-1})\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (2.6)

1- $H=\langle(x,z)\rangle=\langle(x,z^2)\rangle=\langle(x,z^3)\rangle\ \ldots\ \langle(x,z^{p-1})\rangle$

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) or g=(1,z^3)… or g=(1,z^{p-1}) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(1,I)|}\cdot\varphi(g)$$

$$=\frac{4pm}{|C_H(1,I)|}.1=\frac{p.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{p|C_{Q_{2m}}(1)|}{p|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

Since H ∩CL(g)={(1,I),(1,z),(1,z^2) ,(1,z^3),….. ,(1,z^{p-1})}

(i) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)org=(x^m,z^2) or g=(1,z^2) or... or g=(x^m,z^{p-1}) or g=(1,z^{p-1}) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) or...or g= (1,z^{p-1}) and g∈H.

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{4pm}{|C_H(g)|}.1=\frac{p.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{p|C_{Q_{2m}}(1)|}{p|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={g},φ(g)=1

(b)if g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) or… or g=(x^m,z^{p-1}) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{4pm}{|C_H(g)|}.1=\frac{p.4m}{|C_H(g)|}.1=\frac{p|C_{Q_{2m}}(x^m)|}{p|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(i) if g $=\{(x^i,I),(x^i,z),(x^i,z^2\}$,*i≠m,i≠2m* and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{2pm}{|C_H(g)|}(1+1)$$

$$\frac{p.2m}{|C_H(g)|}.(1+1)=\frac{p|C_{Q_{2m}}(q)|}{p|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1}) =1,g=(q,z)=(q,z^2),q∈ Q_{2m} and q≠ x^m ,q≠1

(ii)if g∉H

$\Phi_{(j,2)}(g)=0$ Since H∩CL(g)= ϕ

2- H=$\langle(y,z)\rangle$

={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z), (1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2),...,(1,z^{p-1}),(y,z^{p-1}),(y^2,z^{p-1}),(y^3,z^{p-1})}

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) or…or g=(1,z^{p-1})and g∈H H∩CL(g)={(1,I),(1,z),(1,z^2),... ,(1,z^{p-1})}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{4pm}{4p}.1=m=.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g= (y^2,z) or g=(y^2,z^2)or… g=(y^2,z^{p-1}) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{\left|C_{Q_{2m}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{4pm}{4p}.1=m=\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z^2) or g=(y^3,I_2)or g= (y^3,z) or g= (y^3,z^2) or… or g= (y^3,z^{p-1}) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{\left|C_{Q_{2m}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{2p}{4p}.(1+1)=1=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)=∅

To able the main result, we will solve the following example:

Example (3.2):

To construct Ar(Q_8×C_3) by using the theorem (3.1) we get the following table:

Table (7)

Ar(Q_{30}×C_{11}) =

Γ-classes	[1,I]	[x^{10},I]	[x^6,I]	[x^2,I]	[x^{15},I]	[x^5,I]	[x^3,I]	[x,I]	[y,I]	[1,z]	[x^{10},z]	[x^6,z]	[x^2,z]	[x^{15},z]	[x^5,z]	[x^3,z]	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2	30	1	2	2	2	1	2	2	2	30
$\|C_{Q_{30}}(CL_\alpha)\|$	660	330	330	330	660	330	330	330	22	660	330	330	330	660	330	330	330	22
$\Phi_{(1,1)}$	660	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	220	220	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	132	0	132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	44	44	44	44	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	330	0	0	0	330	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	110	10	0	0	110	110	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	66	0	66	0	66	0	66	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	22	22	22	22	22	22	22	22	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	15	0	0	0	15	0	0	0	1	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	60	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	20	20	0	0	0	0	0	0	0	20	20	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	12	0	12	0	0	0	0	0	0	12	0	12	0	0	0	0	0	0
$\Phi_{(4,2)}$	4	4	4	4	0	0	0	0	0	4	4	4	4	0	0	0	0	0
$\Phi_{(5,2)}$	30	0	0	0	30	0	0	0	0	30	0	0	0	30	0	0	0	0
$\Phi_{(6,2)}$	10	10	0	0	10	10	0	0	0	10	10	0	0	10	10	0	0	0
$\Phi_{(7,2)}$	6	0	6	0	6	0	6	0	0	6	0	6	0	6	0	6	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	0
$\Phi_{(9,2)}$	15	0	0	0	15	0	0	0	1	15	0	0	0	15	0	0	0	1

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