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On Artin's Character Table and Artin's Cokernel of the Group($Q_{2m}\times C_4$) When $m=2^h, h\in Z^+$

حول جدول شواخص ارتين ونواة المصاحبه لـ ارتين للزمرة ($Q_{2m}\times C_4$) $h\in Z^+$, $m=2^h$ عندما

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Abstract--- *The main purpose of this paper is to find Artin's characters table of the group ($Q_{2m}\times C_4$)when $m=2^h$, $h\in Z^+$, which is denoted by $Ar(Q_{2m}\times C_4)$ where Q_{2m} is denoted to Quaternion group and C_4 is the cyclic group of order 4 .Moreover we have found The rational valued characters table and the cyclic decomposition of Artin's cokernel $AC(Q_{2m}\times C_4)$ when $m=2^h$, $h\in Z^+$.*

الملخص

ان الهدف الاساسي من هذا البحث هو ايجاد جدول شواخص ارتن للزمرة ($Q_{2m}\times C_4$) عندما $m=2^h$, $h\in Z^+$ والذي يرمز له بالرمز $Ar(Q_{2m}\times C_4)$ حيث ان Q_{2m} هي الزمرة الرباعية العمومية والزمرة C_4 هي الزمرة الدائرية ذات الرتبة 4 بالاضافة الى ذلك تم ايجاد جدول الشواخص النسبية والتجزئة الدائرية للزمرة ($Q_{2m}\times C_4$) عندما $m=2^h$, $h\in Z^+$.

Key words--- Quaternion group, Artin's characters table, The rational valued characters table, Artin's cokernel, the cyclic subgroup.

1. INTRODUCTION

For a finite group G, let R(G) denote the group generated by z- valued characters of the group G .Inside this group, we have a subgroup generated by Artin's characters (the characters induced from the principal characters of cyclic subgroups) of G which will be denoted by T(G). The factor group R(G)/T(G) which is denoted by AC(G) is called Artin's cokernal of G characters and it is a finite abelain group of the exponent A(G) which is called Artin's exponent. Let x and y be two elements of G, x and y are called Γ-conjugate if the cyclic subgroups which they generate, are Γ-conjugate in G. this is defined at an equivalent relation on G, its classes are called Γ- classes of G.

The square matrix whose rows correspond to Artin's characters and columns correspond to the Γ- classes of G is called Artin's characters table. This matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G).

In 1967 T.Y. lam [11] studied A(G) extensively for many groups In 1981 C.Curits and I. Reiner studied Methods of Representation Theory with application to finite groups. In 2009 S.J. Mahmood [10] studied the general from of Artin's characters table $Ar(Q_{2m})$ when m is an even number.

The aim of this paper is to find the general from of the Artin's characters table and the rational valued characters table and the Artin cokernal of the group ($Q_{2m}\times C_4$) when $m=2^h$,$h\in Z^+$.

2. PRELIMINARIES

This section introduce some important definitions and basic concepts of the group ($Q_{2m}\times C_4$), a rational valued characters, a rational valued characters table, the Artin characters and the Artin characters table.

The Group ($Q_{2m}\times C_4$)(2.1):

The direct product group ($Q_{2m}\times C_4$) where Q_{2m} is Quaternion group of order 4m with two generators x and y is denoted by

$Q_{2m}=\{x^ky^j:x^{2m}=y^4=1,yx^my^{-1}=x^{-m},0\leq k\leq 2m-1,j=0,1\}$

and C_4 is a cyclic group of order 4 consisting of elements $\{I,z,z^2,z^3\}$.

The generalized group ($Q_{2m}\times C_4$) is denoted by

$(Q_{2m}\times C_4)=\{(q,c):q\in Q_{2m},c\in C_4\}$ and $|Q_{2m}\times C_4|=|Q_{2m}|.|C_4|=4m.4=16m$

Definition (2.2):[7]

A rational valued character θ of G is a character whose values are in Z, which is $\theta(g)\in Z$, for all $g\in G$

Corollary (2.3):[8]

The rational valued characters

$\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form the basis for $\overline{R}$(G), where χ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of cyclic subgroup of G .

Theorem (2.4):[3]

Let T_1: $G_1 \rightarrow GL(n,F)$ and T_2: $G_2 \rightarrow GL(m,F)$ be two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively then :

$T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

Proposition (2.5):[6]

The number of all rational valued characters of a finite group G is equal to the number of all distinct Γ- classes on G.

Definition (2.6): [8]

The complete information about rational valued characters of a finite group G is displayed in a table called **rational valued characters table of G**. We refer to it by $\overset{*}{\equiv}(G)$ which is n×n matrix whose columns are Γ-classes and rows which are the values of all rational valued characters of G, where n is the number of Γ-classes.

Proposition (1.7):[9]

The general form of the rational valued characters table of the Quaternion group Q_{2m} when $m=2^h$, h is any positive integer and it is given by:

Table (1)

$\overset{*}{\equiv}(Q_{2m}) = \overset{*}{\equiv}(Q_{2.2^h}) =$

Γ-classes	[1]	$[x^{2^h}]$	$[x^{2^{h-1}}]$	$[x^{2^{h-2}}]$	⋯	$[x^2]$	[x]	[y]	[xy]
θ_1	2^h	-2^h	0	0	- - -	0	0	0	0
θ_2	2^{h-1}	2^{h-1}	-2^{h-1}	0	- - -	0	0	0	0
θ_3	2^{h-2}	2^{h-2}	2^{h-2}	-2^{h-2}	- - -	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮
θ_{l-2}	2	2	2	2	- - -	-2	0	0	0
θ_{l-1}	1	1	1	1	- - -	1	-1	-1	1
θ_l	1	1	1	1	- - -	1	1	-1	-1
θ_{l+1}	1	1	1	1	- - -	1	-1	1	-1
θ_{l+2}	1	1	1	1	- - -	1	1	1	1

Where l is the number of Γ-classes of C_m .

Theorem(2.8): [5]

Let H be a cyclic subgroup of G and $h_1, h_2, \ldots, h_m$ are chosen as representative for m-conjugate classes of H contained in CL(g) in G, then :

1- $\varphi'(g) = \dfrac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$.

Definition(2.9):[11]

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G.** In theorem (1.8) , if φ is the principal character , then $\varphi(h_i) = \varphi(1) = 1$, where $h_i \in H$

Proposition(2.10):[2]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G Furthermore, Artin's characters are constant on each Γ-classes.

Definition(2.11): [1]

Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

Proposition (1.12): [10]

The Artin's characters table of the Quaternion group Q_{2m} when $m=2^h$, $h \in Z^+$ is given by:

Table (2)

$\mathrm{Ar}(Q_2^{h+1}) =$

Γ- classes	Γ- classes of C_{2m}					
	[1]	$[x^{2^h}]$			**[y]**	**[xy]**
$\lvert CL_\alpha \rvert$	**1**	**1**	**2 2 …**	**2**	$\mathbf{2^h}$	$\mathbf{2^h}$
$\lvert C_{Q_{2^{h+1}}}(CL_\alpha)\rvert$	$\mathbf{2^{h+2}}$	$\mathbf{2^{h+2}}$	$\mathbf{2^{h+1}}$ $\mathbf{2^{h+1}}$ **…**	$\mathbf{2^{h+1}}$	**4**	**4**
Φ_1	$2\mathrm{Ar}(C_2^{h+1})$				**0**	**0**
Φ_2					**0**	**0**
⋮					⋮	⋮
Φ_l					**0**	**0**
Φ_{l+1}	$\mathbf{2^h}$	$\mathbf{2^h}$	**0 0 …**	**0**	**2**	**0**
Φ_{l+2}	$\mathbf{2^h}$	$\mathbf{2^h}$	**0 0 …**	**0**	**0**	**2**

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group Q_{2m} when $m=2^h$, $h \in Z^+$

3. THE FACTOR GROUP AC(G)

This section is devoted to the study of the factor group AC(G) of a group G .

Definition(3.1):[8]

Let T(G) be the subgroup of $\overline{R}$(G) generated by Artin's characters .T (G) is normal subgroup of $\overline{R}$(G) and denotes the factor abelian group $\overline{R}$(G)/T(G) by AC(G) which is called **Artin cokernel of** G.

Definition(3.2):[7]

Let M be a matrix with entries in a principal domain R. **A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

Definition(3.3):[7]

A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M.This is denoted by D_k (M)

Lemma(3.4):[7]

Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

Theorem(3.5):[7]

Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1- P and W are invertible.
2- P M W = D.
3- D is diagonal matrix.
4- If we denote D_{ii} by d_i then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d_j = 0$ and $j \le m$ implies $d_j \neq 0$ and $1 \le j \le m$ implies $d_j \mid d_{j+1}$.

Definition (3.6):[7]

Let M be matrix with entries in a principal domain R, be equivalent to a matrix $D = \mathrm{diag}\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j \mid d_{j+1}$ for $1 \le j < m$

We call D **the invariant factor matrix of** M and $d_1, d_2, \dots, d_m$ the invariant factors of M

Theorem(3.7):[7]

Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal $\langle d_1\rangle, \langle d_2\rangle, \dots, \langle d_m\rangle$, $d_j \mid d_{j+1}$ for j = 1, 2, … , K-1.

4.THE MATRIX M(G)

This section is devoted to the study of the matrix M(G), $M(Q_{2m})$,$P(Q_{2m})$ and $W(Q_{2m})$.

Proposition (4.1):[8]

If AC(G) is a finitely generated Z- module and let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l and there exists an invertible matrix M(G) with entries in rational number such that is:

$\equiv^*(G)=M^{-1}(G).\mathrm{Ar}(G)$ and this implies

$M(G)=\mathrm{Ar}(G).(\equiv^*(G))^{-1}$

Theorem (4.2):[4]

$$AC(G) = \bigoplus_{i=1}^{l} C_{d_i} \quad \text{where } d_i = \pm D_i(G) / D_{i-1}(G)$$

where l is the number of all distinct Γ-classes.

Corollary (4.3):[8]

$$|AC(G)| = |\det(M(G))|$$

Proposition (4.4):[10]

If $m=2^h$,h any positive integers ,then the matrix $M(Q_{2m})$ of the quaternion group Q_{2m} is :

$$M(Q_{2m}) = \left[\begin{array}{cccccc|cccc}
2 & 2 & 2 & \cdots & \cdots & \cdots & 1 & 1 & 1 & 1 \\
0 & 2 & 2 & \cdots & \cdots & \cdots & 1 & 1 & 1 & 1 \\
0 & 0 & 2 & \cdots & \cdots & \cdots & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \cdots & 2 & 1 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1
\end{array}\right]$$

which is (h+4)×(h+4) square matrix

Proposition(4.5):[10]

If $m=2^h$,h any positive integer then the matrices $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms:

$$P(Q_{2m}) = \begin{bmatrix}
1 & -1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & -1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & -1 & \cdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & \cdots & 0 & -1 & 1 \\
0 & 0 & \cdots & \cdots & 0 & 0 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 1 & -1 \\
0 & 0 & \cdots & \cdots & 0 & 0 & 1
\end{bmatrix}$$

$$\textit{and} \quad W(Q_{2m}) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\
0 & -1 & -1 & \cdots & -1 & -1 & 0 & 0 & 1
\end{bmatrix}$$

They are (h+4)×(h+4) square matrix .

Example (4.6):

To find $P(Q_{16})$ and $W(Q_{16})$, by the proposition (4.5).

$$P(Q_{16}) = P(Q_{2^4}) = \begin{pmatrix}
1 & -1 & & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$\textit{and} \quad W(Q_{16}) = W(Q_{2^4}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & -1 & -1 & -1 & 0 & 0 & 1
\end{pmatrix}$$

which is 7×7 square matrix

5. THE MAIN RESULTS

In this section we find the general form of Artin's characters and Artin's cokernel of the group ($Q_{2m} \times C_4$) when $m=2^h$, $h \in Z^+$

Proposition(5.1):

The rational valued characters table of the group ($Q_{2m} \times C_4$) when $m=2^h, h \in Z^+$ is equal to the tensor product of the rational valued characters table of Q_{2m} when $m=2^h, h \in Z^+$ and the rational valued characters table of C_4 that is:

$$\overset{*}{\equiv}(Q_{2m} \times C_4) = \overset{*}{\equiv}(Q_{2m}) \otimes \overset{*}{\equiv}(C_4) \ .$$

<u>*Proof*</u> *:-*

$C_4 = \{1, z, z^2, z^3\}$, Since

Table(3)

$\equiv C_4 = \overset{*}{\equiv}(C_4) =$

	h'_1	h'_2	h'_3
χ'_1	2	-2	0
χ'_2	1	1	-1
χ'_3	1	1	1

Where $h'_1 = \{(I)\}$, $h'_2 = \{z^2\}$, $h'_3 = \{z, z^3\}$ then,

$\chi'_1(h'_1) = \theta'_1(h'_1) = 2$

$\chi'_1(h'_2) = \theta'_1(h'_2) = -2$

$\chi'_1(h'_3) = \theta'_1(h'_3) = 0$

$\chi'_2(h'_1) = \chi'_2(h'_2) = \theta'_2(h'_1) = \theta'_2(h'_2) = 1$

$\chi'_2(h'_3) = \theta'_2(h'_3) = -1$

$\chi'_3(h'_1) = \chi'_3(h'_2) = \chi'_3(h'_3) = \theta'_3(h'_1) = \theta'_3(h'_2) = \theta'_3(h'_3) = 1$

From the definition of $Q_{2m} \times C_4$, and Theorem (1.2) we have

$(\equiv Q_{2m}\times C_4) = (\equiv Q_{2m}) \otimes (\equiv C_4)$

Each element in $Q_{2m}\times C_4$

$h_{ng} = h_n \cdot h'_g \quad \forall h_n \in Q_{2m}$ **,** $h'_g \in C_4$ **,**

$n = 1,2,3,\ldots,4m$, $\mathbf{g} \in \{I, z, z^2, z^3\}$**,**

each irreducible character of $Q_{2m}\times C_4$ is

$\chi_{(i,j)} = \chi_i . \chi'_j$

Where χ_i is an irreducible character of Q_{2m}

and χ'_j is the irreducible character of C_4 ,then

$$\chi_{(i,j)}(h_{ng}) = \begin{cases} 2\chi_i(h_n) & if \quad j=1 \ and \ g \in \{I\} \\ -2\chi_i(h_n) & if \quad j=1 \ and \ g \in \{z^2\} \\ 0\chi_i(h_n) & if \quad j=1 \ and \ g \in \{z, z^3\} \\ \chi_i(h_n) & if \quad j=2 \ and \ g \in \{I, z^2\} \\ -\chi_i(h_n) & if \quad j=2 \ and \ g \in \{z, z^3\} \\ \chi_i(h_n) & if \quad j=3 \ and \ g \in C_4 \end{cases}$$

From Proposition (2.4)

$$\theta_{(i,j)} = \sum_{\sigma \in Gal(Q(\chi_{(i,j)})/Q)} \sigma(\chi_{(i,j)})$$

Where $\theta_{(i,j)}$ is the rational valued character of $Q_{2m}\times C_4$

Then,

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)} \sigma(\chi_{(i,j)}(h_{ng}))$$

(I) (a) If j=1 and $g \in \{I\}$

$$\theta_{(i,j)}(h_{ng}) = \theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(2\chi_i(h_n)) =$$

$$2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 2$$

$$= \theta_i(h_n) \,.\, \theta'_j(h'_g)$$

(b) If j=1 and $g \in \{z^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(-2\chi_i(h_n)) =$$

$$-2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \theta_i(h_n) \,.\, -2 = \theta_i(h_n) \,.\, \theta'_j(h'_g)$$

(c) if j=1 and $g \in \{z, z^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(0\chi_i(h_n)) = 0.$$

$$\sum_{\sigma \in Gal(Q(\chi_I(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 0 = \theta_i(h_n) \,.\, \theta'_j(h'_g)$$

Where θ_i is the rational valued character of Q_{2m} .

(II) (a) If j=2 and $g \in \{I, z^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 1$$

$$= \theta_i(h_n) \,.\, \theta'_j(h'_g)$$

(b) If j=2 and $g \in \{z, z^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n)) =$$

$$- \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) \,.\, -1 = \theta_i(h_n) \,.\, -1 =$$

$$\theta_i(h_n) \,.\, \theta'_j(h'_g) \,.$$

(III) If j=3 and $g \in C_4$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n) \,.\, 1$$

$$= \theta_i(h_n) \,.\, \theta'_j(h'_g)$$

From [I] , [II] and [III] we have

$$\theta_{(i,j)} = \theta_i \,.\, \theta'_j \,.$$

Then $\equiv^*(Q_{2m}\times C_4) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_4)$.

Example (5.2)

To find $\equiv^*(Q_{16}\times C_4)$ by using the Proposition (5.1) we get the following table:

$\equiv^*(Q_2{}^{16}\times C_4)=$

Table(4)

Γ-classes	[1,I]	[x^8,I]	[x^4,I]	[x^2,I]	[x,I]	[y,I]	[xy,I]	[1,z^2]	[x^8,z^2]	[x^4,z^2]	[x^2,z^2]	[x,z^2]	[y,z^2]	[xy,z^2]	[1,z]	[x^8,z]	[x^4,z]	[x^2,z]	[x,z]	[y,z]	[xy,z]
$\theta_{(1,1)}$	16	-16	0	0	0	0	0	-16	16	0	0	0	0	0	0	0	0	0	0	0	0
$\theta_{(2,1)}$	8	8	-8	0	0	0	0	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0
$\theta_{(3,1)}$	4	4	4	-4	0	0	0	-4	-4	-4	4	0	0	0	0	0	0	0	0	0	0
$\theta_{(4,1)}$	2	2	2	2	-2	-2	2	-2	-2	-2	-2	2	2	-2	0	0	0	0	0	0	0
$\theta_{(5,1)}$	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	2	2	0	0	0	0	0	0	0
$\theta_{(6,1)}$	2	2	2	2	-2	2	-2	-2	-2	-2	-2	2	-2	2	0	0	0	0	0	0	0
$\theta_{(7,1)}$	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0
$\theta_{(1,2)}$	8	-8	0	0	0	0	0	8	-8	0	0	0	0	0	-8	8	0	0	0	0	0
$\theta_{(2,2)}$	4	4	-4	0	0	0	0	4	4	-4	0	0	0	0	-4	-4	4	0	0	0	0
$\theta_{(3,2)}$	2	2	2	-2	0	0	0	2	2	2	-2	0	0	0	-2	-2	-2	2	0	0	0
$\theta_{(4,2)}$	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	1	-1
$\theta_{(5,2)}$	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1
$\theta_{(6,2)}$	1	1	1	1	-1	1	-1	1	1	1	1	-1	1	-1	-1	-1	-1	-1	1	-1	1
$\theta_{(7,2)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
$\theta_{(1,3)}$	8	-8	0	0	0	0	0	8	-8	0	0	0	0	0	8	-8	0	0	0	0	0
$\theta_{(2,3)}$	4	4	-4	0	0	0	0	4	4	-4	0	0	0	0	4	4	-4	0	0	0	0
$\theta_{(3,3)}$	2	2	2	-2	0	0	0	2	2	2	-2	0	0	0	2	2	2	-2	0	0	0
$\theta_{(4,3)}$	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1
$\theta_{(5,3)}$	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1
$\theta_{(6,3)}$	1	1	1	1	-1	1	-1	1	1	1	1	-1	1	-1	1	1	1	1	-1	1	-1
$\theta_{(7,3)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Proposition (5.3):
The general from of the Artin's characters table of the group $(Q_{2^{h+1}}\times C_4)$ when $m=2^h$, $h\in Z^+$ is give as follows:

Table(5)

$Ar(Q_{2^{h+1}}\times C_4)$=

	Γ- classes of $(Q_{2m})\times\{I\}$						Γ- classes of $(Q_{2m})\times\{z^2\}$						Γ- classes of $(Q_{2m})\times\{z\}$					
Γ- classes	$[1,I]$	$[x^m,I]$	…	$[x,I]$	$[y,I]$	$[xy,I]$	$[I,z^2]$	$[x^m,z^2]$	…	$[x,z^2]$	$[y,z^2]$	$[xy,z^2]$	$[I,z]$	$[x^m,z]$	…	$[x,z]$	$[y,z]$	$[xy,z]$
$\lvert CL_\alpha\rvert$	1	1	…	2	m	m	1	1	…	2	m	m	1	1	…	2	m	m
$\lvert C_{Q_{2m}\times C_4}(CL_\alpha)\rvert$	16m	16m	…	8m	16	16	16m	16m	...	8m	16	16	16m	16m	...	8m	16	16
$\Phi_{(1,1)}$, $\Phi_{(2,1)}$, ⋮, $\Phi_{(l,1)}$, $\Phi_{(l+1,1)}$, $\Phi_{(l+2,1)}$	**$4Ar(Q_{2m})$**						**0**						**0**					
$\Phi_{(1,2)}$, $\Phi_{(2,2)}$, ⋮, $\Phi_{(l,2)}$, $\Phi_{(l+1,2)}$, $\Phi_{(l+2,2)}$	**$2Ar(Q_{2m})$**						**$2Ar(Q_{2m})$**						**0**					
$\Phi_{(1,3)}$, $\Phi_{(2,3)}$, ⋮, $\Phi_{(l,3)}$, $\Phi_{(l+1,3)}$, $\Phi_{(l+2,3)}$	**$Ar(Q_{2m})$**						**$Ar(Q_{2m})$**						**$Ar(Q_{2m})$**					

Proof :

Let $g\in(Q_{2m}\times C_4)$;g=(q,I) or g=(q,z)or g=(q,z²)or g=(q,z³) $q\in Q_{2m}$,I,z,z²,z³∈ C_4

Case (I):

If H is a cyclic subgroup of $Q_{2m}\times\{I\}$,then:

1.H=⟨(x, I)⟩

2. H=⟨(y, I)⟩

3. H=⟨(xy, I)⟩

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (2.8)

1. H=⟨(x, I)⟩

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I))=\frac{\lvert C_{Q_{2m}\times C_4}(g)\rvert}{\lvert C_H(g)\rvert}\cdot\varphi(g)$$

$$=\frac{16m}{\lvert C_H(I,1)\rvert}.1=\frac{4.4m}{\lvert C_H(I,1)\rvert}.1=\frac{4\lvert C_{Q_{2m}}(1)\rvert}{\lvert C_{\langle x\rangle}(1)\rvert}.\varphi(1)=4.\Phi_j(1)$$

since H ∩CL(1,I)={(1,I)}

(ii) if g= (x^m,I) and g∈H

$$\Phi_{(j,1)}(g)=\frac{\lvert C_{Q_{2m}\times C_4}(g)\rvert}{\lvert C_H(g)\rvert}\cdot\varphi(g)$$

$$=\frac{16m}{\lvert C_H(g)\rvert}.1=\frac{4.4m}{\lvert C_H(g)\rvert}.1=\frac{4\lvert C_{Q_{2m}}(x^m)\rvert}{\lvert C_{\langle x\rangle}(x^m)\rvert}.\varphi(g)=4.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i,I),i\ne m$ and $i\ne 2m$ and g∈H

$$\Phi_{(j,1)}(g)=\frac{\lvert C_{Q_{2m}\times C_4}(g)\rvert}{\lvert C_H(g)\rvert}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{\lvert C_H(g)\rvert}(1+1)=$$

$$\frac{4.2m}{\lvert C_H(g)\rvert}.(1+1)=\frac{4\lvert C_{Q_{2m}}(q)\rvert}{\lvert C_{\langle x\rangle}(q)\rvert}.(\varphi(g)+\varphi(g^{-1}))=4.\Phi_j(q)$$

since H∩CL(g)={g,g⁻¹} and φ(g)=φ(g⁻¹)=1,g=(q,I),q∈ Q_{2m} and $q\ne x^m$,q≠1

(iv) if g∉ H

$$\Phi_{(j,1)}(g)=4.0=4.\Phi_j(q)$$

Since H∩CL(g)= ϕ

2. H=⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,1)}(g)=0 \quad \text{since H∩CL(g)=∅}$$

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),($(xy)^2$,I),($(xy)^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{4}.1=4m=4.\Phi_{l+2}(x^m)$$

$\Phi_{(j,2)}(g)=2.0=2.\Phi_j(q)$ Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=($(xy)^3$,I) =(xy^3,I) and g∈H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+2,1)}(g)=0 \quad \text{since H∩CL(g)=∅}$$

Case (II):

If H is a cyclic subgroup of $Q_{2m}\times$ {z^2},then:

1.H=⟨(x, I)⟩=$\langle (x,z^2\rangle$

2. H=⟨(y, I)⟩ = $\langle (y,z^2\rangle\rangle$

3. H=⟨(xy, I)⟩ = $\langle (xy,z^2\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (2.8)

1. H=⟨(x, I)⟩ = $\langle (x,z^2\rangle$

(i) If g=(1,I) or g=(1,z^2) and g∈H

$$\Phi_{(j,2)}((1,I))=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(I,1)|}.1=\frac{4.4m}{|C_H(I,1)|}.1=\frac{4|C_{Q_{2m}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=2.\Phi_j(1)$$ since

H∩CL(1,I)={(1,I),(1,z^2)}

(ii) if g= (x^m, I) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{2|C_{\langle x\rangle}(x^m)|}.\varphi(g)=2.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i, I), i\ne m$ and $i\ne 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)=$$

$$\frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{2|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g∉ H

Since H∩CL(g)= ϕ

2. H=⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I), (1, z^2),(y, z^2),(y^2, z^2),(y^3,z^2)}

(i) If g=(1,I) or g=(1,z^2)

H∩CL(1,I)={(1,I), (1,z^2)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g=(y^2,z^2) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) or g=(y,z^2) or g=(y^3, z^2) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)=∅

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),($(xy)^2$,I),($(xy)^3$,I), (1, z^2),(xy, z^2),($(xy)^2$, z^2),($(xy)^3$,z^2)}

(i) If g=(1,I) or g=(1,z^2) H∩CL(1,I)={(1,I),(1,z^2)}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(1)$$

(ii)If g= (x^m, I)=($(xy)^2$,I)=(y^2,I) or g= (x^m, z^2)=($(xy)^2$,z^2)=(y^2,z^2) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{8}.1=2m=2.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(ii) If g=(xy,I) or g=($(xy)^3$,I) =(xy^3,I) or g=(xy,z^2) or g=($(xy)^3$, z^2) =(xy^3, z^2) and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,2)}(g)=0$ since H∩CL(g)=∅

<u>Case (III):</u>

If H is a cyclic subgroup of (Q_{2m}× {z}),then:

1.H=$\langle(x,z)\rangle=\left\langle(x,z^2)\right\rangle=\left\langle(x,z^3)\right\rangle$

2. H=$\langle(y,z)\rangle=\left\langle(y,z^2)\right\rangle=\left\langle(y,z^3)\right\rangle$

3. H=$\langle(xy,z)\rangle=\left\langle(xy,z^2)\right\rangle=\left\langle(xy,z^3)\right\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.8)

1.H=$\langle(x,z)\rangle$

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) or g=(1,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(1,I)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(1,I)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),(1,z^2),(1,z^3)}

(ii) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)or g=(x^m,z^2) or g=(1,z^2)or g=(1,z^3) or g=(x^m,z^3) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3) and g∈H.

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={g},φ(g)=1

(b)If g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) or g=(x^m,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{4|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) If g$=\{(x^i,I),(x^i,z),(x^i,z^2),(x^i,z^3)\}$,$i\neq m,i\neq 2m$ and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)$$

$$\frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{4|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z)=(q,z^3),q∈ Q_{2m} and q≠ x^m ,q≠1

(iii) if g∉H

$\Phi_{(j,)}(g) = 0$ Since H∩CL(g)= ϕ

2. H=$\langle (y,z) \rangle$

={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z),(1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2),(1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3))}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3)and g∈H

H∩CL(g)={(1,I),(1,z),(1,z^2) ,(1,z^3)}

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = .\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g= (y^2,z) or g=(y^2,z^2) or g=(y^2,z^3) and g∈ H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z^2) or g=(y,z^3) or g=(y^3,I_2)or g= (y^3,z) or g= (y^3,z^2) or g= (y^3,z^3) and g∈H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{16}{16}.(1+1) = 2 = \Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,3)}(g) = 0$ since H∩CL(g)=∅

3.H=$\langle (xy,z) \rangle$

={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I),(1,z) ,(xy,z), ($(xy)^2$,z)),($(xy)^3$,z),(1,z^2),(xy,z^2),($(xy)^2$,z^2)),($(xy)^3$,z^2) ,(1,z^3) (xy,z^3),($(xy)^2$,z^3)),($(xy)^3$,z^3)}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3) H∩CL(g)={g}

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=($(xy)^2$,I)=(y^2,I)or g=($(xy)^2$,z)=(y^2,z)or g=($(xy)^2$,z^2)=(y^2,z^2) or g=($(xy)^2$,z^3)=(y^2,z^3) and g∈ H

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{16}.1 = m = \Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g= ($(xy)^3$,I) or g= (xy,z) or g= ($(xy)^3$,z) or g= (xy,z^2) or

(iv) g= ($(xy)^3$,z^2) or g= (xy,z^3) or g= ($(xy)^3$,z^3) and g∈H

$$\Phi_{(l+2,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{16}{16}.(1+1) = 2 = \Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,3)}(g) = 0$ since H∩CL(g)=∅

Example (5.4):

To construct Ar(Q_{16}×C_4) by using the theorem (5.1) we get the following table:

Ar(Q_2^{16}×C_4)=

Table (6)

-classes	[1,I]	$[x^8,I]$	$[x^4,I]$	$[x^2,I]$	[x,I]	[y,I]	[xy,I]	$[1,z^2]$	$[x^8,z^2]$	$[x^4,z^2]$	$[x^2,z^2]$	$[x,z^2]$	$[y,z^2]$	$[xy,z^2]$	[1,z]	$[x^8,z]$	$[x^4,z]$	$[x^2,z]$	[x,z]	[y,z]	[xy,z]
$\|CL_\alpha\|$	1	1	2	2	2	8	8	1	1	2	2	2	8	8	1	1	2	2	2	8	8
$\|C_{Q_{2^7}}(CL_\alpha)\|$	128	128	64	64	64	16	16	128	128	64	64	64	16	16	128	128	64	64	64	16	16
$\Phi_{(1,1)}$	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	64	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	32	32	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	32	32	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	32	32	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	64	0	0	0	0	0	0	64	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	32	32	0	0	0	0	0	32	32	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	16	16	16	0	0	0	0	16	16	16	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	8	8	8	8	0	0	0	8	8	8	8	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,2)}$	4	4	4	4	4	0	0	4	4	4	4	4	0	0	0	0	0	0	0	0	0
$\Phi_{(6,2)}$	16	16	0	0	0	4	0	16	16	0	0	0	4	0	0	0	0	0	0	0	0
$\Phi_{(7,2)}$	16	16	0	0	0	0	4	16	16	0	0	0	0	4	0	0	0	0	0	0	0
$\Phi_{(1,3)}$	32	0	0	0	0	0	0	32	0	0	0	0	0	0	32	0	0	0	0	0	0
$\Phi_{(2,3)}$	16	16	0	0	0	0	0	16	16	0	0	0	0	0	16	16	0	0	0	0	0
$\Phi_{(3,3)}$	8	8	8	0	0	0	0	8	8	8	0	0	0	0	8	8	8	0	0	0	0
$\Phi_{(4,3)}$	4	4	4	4	0	0	0	4	4	4	4	0	0	0	4	4	4	4	0	0	0
$\Phi_{(5,3)}$	2	2	2	2	2	0	0	2	2	2	2	2	0	0	2	2	2	2	2	0	0
$\Phi_{(6,3)}$	8	8	0	0	0	2	0	8	8	0	0	0	2	0	8	8	0	0	0	2	0
$\Phi_{(7,3)}$	8	8	0	0	0	0	2	8	8	0	0	0	0	2	8	8	0	0	0	0	2

Proposition (5.5):

If $m=2^h$,h any positive integer then the matrix $M(Q_{2m}\times C_4)$ of the group $(Q_{2m}\times C_4)$ is :

$$M(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right]$$

Which is $3(h+4)\times 3(h+4)$ square matrix ,$M(Q_{2m})$ is similar to the matrix in Proposition (4.7).

Proof :

By Proposition (5.3) we obtain the Artin's characters Table $Ar(Q_{2m}\times C_4)$ of the group $(Q_{2m}\times C_4)$ when $m=2^h, h\in Z^+$ and from the Proposition (5.1) we get the rational valued characters table $(\overset{*}{\equiv}(Q_{2m}\times C_4))$ of the group $(Q_{2m}\times C_4)$ when $m=2^h, h\in Z^+$.

Thus, by definition of M(G) we can find the matrix $M(Q_{2m}\times C_4)$ when $m=2^h, h\in Z^+$.

$$M(Q_{2m}\times C_4)=Ar(Q_{2m}\times C_4)\cdot(\overset{*}{\equiv}(Q_{2m}\times C_4))^{-1}.$$

$$=\left[\begin{array}{cccccccccccccccccccccccccccccc}
2&2&2&\cdots&\cdots&2&1&1&1&1&2&2&2&\cdots&\cdots&2&1&1&1&1&2&2&2&\cdots&\cdots&2&1&1&1&1\\
0&2&2&\cdots&\cdots&2&1&1&1&1&0&2&2&\cdots&\cdots&2&1&1&1&1&0&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&2&\cdots&\cdots&2&1&1&1&1&0&0&2&\cdots&\cdots&2&1&1&1&1&0&0&2&\cdots&\cdots&2&1&1&1&1\\
\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots\\
0&0&0&\cdots&\cdots&2&1&1&1&1&0&0&0&\cdots&\cdots&2&1&1&1&1&0&0&0&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&1&1&1&1&0&0&0&\cdots&\cdots&0&1&1&1&1&0&0&0&\cdots&\cdots&0&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&1&1&1&0&0&0&\cdots&\cdots&0&0&1&1&1&0&0&0&\cdots&\cdots&0&0&1&1&1\\
0&1&1&\cdots&\cdots&1&0&0&0&1&0&1&1&\cdots&\cdots&1&0&0&0&1&0&1&1&\cdots&\cdots&1&0&0&0&1\\
0&1&1&\cdots&\cdots&1&1&0&1&1&0&1&1&\cdots&\cdots&1&1&0&1&1&0&1&1&\cdots&\cdots&1&1&0&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&2&2&2&\cdots&\cdots&2&1&1&1&1&2&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&2&2&\cdots&\cdots&2&1&1&1&1&0&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&2&\cdots&\cdots&2&1&1&1&1&0&0&2&\cdots&\cdots&2&1&1&1&1\\
\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&2&1&1&1&1&0&0&0&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&1&1&1&1&0&0&0&\cdots&\cdots&0&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&1&1&1&0&0&0&\cdots&\cdots&0&0&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&1&1&\cdots&\cdots&1&0&0&0&1&0&1&1&\cdots&\cdots&1&0&0&0&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&1&1&\cdots&\cdots&1&1&0&1&1&0&1&1&\cdots&\cdots&1&1&0&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&2&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&2&\cdots&\cdots&2&1&1&1&1\\
\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&1&1&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&1&1&\cdots&\cdots&1&0&0&0&1\\
0&0&0&\cdots&\cdots&0&0&0&0&0&0&0&0&\cdots&\cdots&0&0&0&0&0&0&1&1&\cdots&\cdots&1&1&0&1&1
\end{array}\right]$$

$$=\left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right]=M(Q_{2m}\times C_4)$$

Example (5.6):

Consider the group $(Q_{16}\times C_4)$, we can find the matrix $M(Q_{16}\times C_4)$ by using:

$$M(Q_{16}\times C_4)=M(Q_{2^4}\times C_4)=Ar(Q_{2^4}\times C_4)\cdot(\overset{*}{\equiv}(Q_{2^4}\times C_4))^{-1}$$

$$=\begin{bmatrix}
2&2&2&1&1&1&1&2&2&2&1&1&1&1&2&2&2&1&1&1&1\\
0&2&2&1&1&1&1&0&2&2&1&1&1&1&0&2&2&1&1&1&1\\
0&0&2&1&1&1&1&0&0&2&1&1&1&1&0&0&2&1&1&1&1\\
0&0&0&1&1&1&1&0&0&0&1&1&1&1&0&0&0&1&1&1&1\\
0&0&0&0&1&0&1&0&0&0&0&1&0&1&0&0&0&0&1&0&1\\
0&1&1&0&0&1&1&0&1&1&0&0&1&1&0&1&1&0&0&1&1\\
0&1&1&1&0&0&1&0&1&1&1&0&0&1&0&1&1&1&0&0&1\\
0&0&0&0&0&0&0&2&2&2&1&1&1&1&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&2&2&1&1&1&1&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&2&1&1&1&1&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&1&1&0&0&1&1&0&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&1&1&1&0&0&1&0&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&1
\end{bmatrix}$$

Proposition (5.7):

If $m=2^h$, h any positive integers then the matrices $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ are taking the forms :

$$P(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} P(Q_{2m}) & -P(Q_{2m}) & 0 \\ \hline 0 & P(Q_{2m}) & -P(Q_{2m}) \\ \hline 0 & 0 & P(Q_{2m}) \end{array}\right]$$

which is $3(h+4)\times 3(h+4)$ square matrix .

And

$$W(Q_{2m}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{2m}) & 0 & 0 \\ \hline 0 & W(Q_{2m}) & 0 \\ \hline 0 & 0 & W(Q_{2m}) \end{array}\right]$$

Which is $3(h+4)\times 3(h+4)$ square matrix .

Proof :

By using the proposition (5.5) taking the matrix $M(Q_{2m}\times C_4)$ and the above forms $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ then we have :

$P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).W(Q_{2m}\times C_4)$ = diag $\{\underbrace{2,2,2,2,\cdots,2}_{3(h+1)},1,1,1,1,1,1,1,1,1\}$

$=D(Q_{2m}\times C_4)$

which is $3(h+4)\times 3(h+4)$ square matrix .

Example (5.8):

To find the matrices $P(Q_{16}\times C_4)$ and $W(Q_{16}\times C_4)$ by the proposition (6.7) from Example (4.6) to find $P(Q_{16})$ and $W(Q_{16})$:

$$P(Q_{16}\times C_4)=\left[\begin{array}{c|c|c} P(Q_{16}) & -P(Q_{16}) & 0 \\ \hline 0 & P(Q_{16}) & -P(Q_{16}) \\ \hline 0 & 0 & P(Q_{16}) \end{array}\right]=$$

$$\begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

And

$$W(Q_{16}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{16}) & 0 & 0 \\ \hline 0 & W(Q_{16}) & 0 \\ \hline 0 & 0 & W(Q_{16}) \end{array}\right]=$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 1
\end{pmatrix}$$

Example (5.9)

To find $D(Q_{16}\times C_4)$ and the cyclic decomposition of the factor group

We find the matrices $P(Q_{16}\times C_4)$ and $W(Q_{16}\times C_4)$ as in example (5.8) and $M(Q_{16}\times C_4)$ as in example (5.6),then :

$P(Q_{16}\times C_4).M(Q_{16}\times C_4).W(Q_{16}\times C_4)=$

diag{2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1}=

$D(Q_{16}\times C_4)$

Then by Theorem (4.2) we have

$$AC(D(Q_{16}\times C_4))=\bigoplus_{i=1}^{12} C_2$$

The following theorem gives the cyclic decomposition of the factor group $AC(D(Q_{2m}\times C_4))$ when $m=2^h$, $h\in Z^+$.

Theorem(5.10):

If $m=2^h$, h any positive integer then the cyclic decomposition of $AC(Q_{2m}\times C_4)$ is :

$$AC(D(Q_{2m}\times C_4))=\bigoplus_{i=1}^{3(h+1)} C_2$$

<u>Proof :</u>

By using the proposition (5.5),we can find matrix $M(Q_{2m}\times C_4)$ and by the proposition (5.7),we find $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ when $m=2^h$, $h\in Z^+$:

$P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).\ W(Q_{2m}\times C_4)=$

diag{2,2,2,2,2,2,…,2,2,2,1,1,1,1,1,1,1,1,1}

Then ,by the theorem (4.2) we have :

$$AC(D(Q_{2m}\times C_4))=\bigoplus_{i=1}^{3(h+1)} C_2$$

Example(5.11) :

Consider the groups $(Q_{32768}\times C_4)$,$(Q_{16777216}\times C_4)$ then :

1. $AC(Q_{32768}\times C_4)=AC(Q_{2^{15}}\times C_4)=\bigoplus_{i=1}^{45} C_2$
2. $AC(Q_{16777216}\times C_4)=AC(Q_{2^{24}}\times C_4)=\bigoplus_{i=1}^{72} C_2$

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