University of Kufa
College of Education for Girls
Department of Mathematics

On Artin Cokernel of The Quaternion Group Q_{2m} When m is an Odd Number

A Thesis Submitted to
**The Council of the College of Education for Girls **
University of Kufa
In Partial Fulfillment of the Requirements for the Master Degree in Mathematics

By
Atika Hassan Abdul-Mun'em Shubeil AL-Humeidawi

Supervised by
Asst. Prof. Nesir Rasool Mahmood

1429A.H. **2008A.D.**

بسم الله الرحمن الرحيم

﴿ اللَّهُ نُورُ السَّمَاوَاتِ وَالأَرْضِ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحٌ الْمِصْبَاحُ فِي زُجَاجَةٍ الزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ يُوقَدُ مِنْ شَجَرَةٍ مُبَارَكَةٍ زَيْتُونَةٍ لاَ شَرْقِيَّةٍ وَلاَ غَرْبِيَّةٍ يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ نُورٌ عَلَى نُورٍ يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ وَيَضْرِبُ اللَّهُ الأَمْثَالَ لِلنَّاسِ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ ﴾

صدق الله ألعلي العظيم

سورة النور إيه (٣٥)

Abstract

The main purpose of this thesis is to determine the cyclic decomposition of the finite abelian factor group $AC(G)=\overline{R}(G)/T(G)$ where $G = Q_{2m}$ and m is an odd number, Q_{2m} is the Quaternion group of order 4m (the group of all Z-valued generalized characters of G over the group of induced unit characters from all cyclic subgroups of G) .

We find that the cyclic decomposition of $AC(Q_{2m})$ depends on the elementary divisor of m , if $m = p_1^{\alpha_1}.p_2^{\alpha_2}..p_n^{\alpha_n}$, where $p_1,p_2,\ldots, p_n$ are distinct primes and $\alpha_1,\alpha_2,..,\alpha_n$ are positive integers, then

$$AC(Q_{2m}) = \bigoplus_{i=1}^{2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_n+1)-2} C_2$$

Moreover , we have also found the general form of Artin's characters table $Ar(Q_{2m})$ when m is an odd number .

We have used the Matlab program to calculate some results of this thesis .

List of Symbols

Symbol	The definition
F	The field.
GL(l,F)	Group of invertible $l \times l$ matrix over the field F.
V	Vector space over the field F.
S_n	The symmetric group of order n!.
T_l	The matrix representation of degree l .
C	The field of complex numbers.
χ	The character of the representation T.
tr(A)	Trace of matrix A .
$\|G\|$	The order of the group G.
$\oplus$	The direct sum.
Irr(G)	The set of all irreducible characters of a group G.
$\|C_G(g)\|$	The order of centralizer of g in G.
CL(g)	The conjugacy class of the element g.
$\equiv$(G)	The characters table of the group G .
C_G(CL)	The centralizer of the conjugacy classes in G .
C_n	The cyclic group of order n.
Q_{2m}	The Quaternion group of order 4m .
θ(g)	The rational valued character of g.
Z	The set of integer numbers.
$\langle x \rangle$	The cyclic group generated by x .

ε_n	The nth-root of unity .
Q	The field of the rational numbers .
Γ	The Galois group $Gal(Q(\varepsilon_n)/Q)$.
$[x]$	The Γ- class of the element x .
$\overset{*}{\equiv}(G)$	The rational valued characters table.
$\chi_n^{\sigma m}$	The elements of Galois group.
$\otimes$	Tensor (kronecker)product.
[G:H]	The index of H in G .
Ar(G)	Artin characters table of the group G .
I_k	The identity square k×k matrix .
D(G)	The invariant factor matrix of the group G.
AC(G)	Artin cokernel of the group G .
$\varphi'(g)$	The value of Artin character of g .
$\Phi(g)$	The value of Artin character of the quaternion group .
$\mid$	Divided by .
T(G)	The group generated by Artin characters .
det(A)	The determinant of matrix A.
diag$\{d_1,\ldots,d_n\}$	The diagonal matrix of elements $d_1,\ldots,d_n$.
ϕ	The empty set .
$\bullet$	The ordinary product.
g.c.d	The greatest common divisor .

List of Tables

Contents